ADVERSE SELECTION AND MORAL HAZARD IN RENTAL HOUSING MARKETS

By

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Daniel Robert LeClair

This dissertation is dedicated to my wife, Loraine, and our dog, Boo.

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In this dissertation, I study adverse selection and moral hazard in rental housing markets. I do this by introducing three original theoretical models of rental housing markets characterized by asymmetric information.

In the first model, tenants possess private information about their transactions costs of moving. A tenant will be bad, put forth zero effort to maintain the unit he occupies, if it is cheaper to move at the end of the period than to maintain the unit. If the cost of effort increases in quality, the proportion of bad tenants increases as quality increases. In equilibrium, quality is "rationed" to good tenants, who put forth effort to maintain the rental unit, and good tenants are separated from bad tenants. Quality rationing occurs when landlords quote an implicit price per unit of quality, but offer lower quality than that demanded by tenants at that price.

In the second model, tenant effort and landlord maintenance affect the quality of a rental unit over time, but effort and maintenance are not contractible.

In a finite-horizon model, equilibrium is inefficient. However, if the landlord-tenant relationship continues over an infinite horizon, it is shown that there exists a simple rental contract which supports Pareto-efficient choices in a subgame perfect equilibrium. This is because tenants and landlords employ "trigger strategies", where efficient effort and maintenance are chosen unless either the landlord or tenant has deviated from these choices in the past. If a deviation does occur, quality will move off the efficient path and the landlord and tenant will revert to less efficient subgame perfect equilibrium choices.

In the third model, the landlord-tenant relationship lasts only one period. With only tenant moral hazard, simple damage deposits can preserve efficiency in equilibrium. When extended to double moral hazard, Pareto-efficiency cannot obtain in equilibrium. However, under certain assumptions regarding the relative cost of effort versus maintenance, it is established that damage deposits can play a positive and effective role in supporting a more efficient equilibrium than would otherwise occur. Also, in equilibrium, quality and economic profit each period decreases over time.

CHAPTER 1
INTRODUCTION TO MODELS OF RENTAL HOUSING MARKETS, ADVERSE SELECTION, AND MORAL HAZARD

1.1 Introduction

Rental housing markets have received surprisingly little attention from economists. This is surprising because of the importance of housing in our economy. For instance, the cost of housing services and household operations comprises almost 40% of the Consumer Price Index. In 1985 almost 40% of all housing in the United States was rental in nature. There was formerly a very large role for rent "ceilings" in many large cities. Housing subsidies continue to be offered to both developers and low-income households. A large proportion of lower-income households occupy rental housing units. Of course, these observations do not imply that rental housing markets have been neglected in the literature. They only suggest that rental housing markets deserve special attention by economists. It will become evident in Section 1.2 that, at least with respect to some issues, economists have not accommodated this need for special attention.

Despite recent developments in theoretical models with imperfect information, there has been only limited discussion of the role of informational problems in rental housing markets. In the past two decades, economists have devoted much attention toward understanding the role of information in a variety of market environments. Most of this attention has focused on the labor, credit, and insurance markets. This literature provides a rich source for studying problems associated with information in rental housing markets. In Sections 1.2

and 1.3, I call attention to some connections between these markets and rental housing markets. In fact, in the series of essays comprising Chapters 2 through 4 of this dissertation, I will develop models to explain behavior in rental housing markets in the context of informational problems, initially discussed with respect to these other markets.

In Section 1.2, I will survey the literature concerned with theoretical models of rental housing. In Section 1.3, I will introduce the problems of adverse selection and moral hazard by reviewing some very simple models and appealing to some of the existing literature. The intent is not to provide an exhaustive survey of the rental housing and asymmetric information literatures in these respective sections. The purpose is instead to provide a selective introduction to the models, ideas, and problems relevant to the models employed in this dissertation. In Section 1.4, I provide an introduction to the models which make up the bulk of this work.

1.2: Survey of Rental Housing Literature

Housing markets have many special characteristics. First, a dwelling is durable, fixed spatially, and different from every other dwelling. Second, there tends to be a great deal of government involvement in housing markets. Through monetary and fiscal policy the government can affect the demand and supply of housing. The government often offers subsidies to households to help cover the costs of housing or to developers to help finance new construction. There are also tax deductions to homeowners. Finally, housing markets are characterized by many informational problems. There is uncertainty concerning the future and substantial asymmetries in information between the players involved in housing markets. It is the latter problem which concerns this dissertation.

Each of these special characteristics is not unique to housing markets. However, it would be difficult to discover another market which possesses all of them. It would also be impossible to incorporate all of these characteristics in one general model of housing markets. Usually, a model will focus on one or two of these characteristics while effectively ignoring the rest. In this dissertation, I will focus on certain asymmetries in information in rental housing markets, often at the expense of the other characteristics. I will now, however, discuss some of the relevant literature concerning rental housing markets and models which incorporate these other characteristics.

Early housing market models did not effectively deal with the complications of durability, heterogeneity, and space. For instance, the heterogeneity of housing was dealt with by assuming the existence of an unobservable homogeneous commodity called housing services (Muth, 1960; Olsen, 1969). The durability of housing was dealt with by assuming that there exists one homogeneous unit of housing stock per unit of time, with perfect capital markets, no taxes, and with asset markets in equilibrium (Deaton and Muellbauer, 1980). In these models, housing services and housing stock may be used interchangeably and households are indifferent between obtaining housing services in the ownership or the rental market. Of course, with the restrictions of these models, there is no insight which can be gained with respect to the above complications. Even though this is the case, many economists have used similar models to study particular issues concerning the rental housing market alone. For instance, Frankena (1975) and Fallis and Smith (1984) study rent controls and the effects of housing allowances on renters. The point is that incorporating these complexities often does not add additional insight into the problem being examined.

These models were later enriched to deal with particular housing characteristics. The effect was to gain not only some realism, but also to gain insight into some very important issues.

Generally, to deal with the problem of durability, housing markets were viewed in a stock-flow manner not unlike the way that economists have examined investment in the capital stock (Olsen, 1969; Muth, 1960; Duesenberry, 1958). For example, in the short run there is a perfectly inelastic supply of housing stock/services and the price is determined by demand. Note that these models make considerable use of dynamic optimization techniques. This is important because in the last two models of this dissertation, I take advantage of this connection.

One line of research of interest for this dissertation concerns tenure choice. These models focus on what factors influence the household decision to rent or own and, thus, hope to explain the extensive renting and owner markets for housing in the United States. One explanation derives from the lower transactions cost of changing rental apartments than of changing owned houses. Because younger people are more likely occupy entry-level jobs or may be students, they tend to have higher probabilities of moving. Thus, younger people tend to rent. Transactions costs will be an important factor in generating the model presented in Chapter 2. Other models direct attention to the tax-advantages of owner-occupancy, mortgage market constraints, the inability of landlords to charge marginal costs (Henderson and Ionnides, 1983), and intertemporal preferences. One interesting idea posits that the choice between owning and renting is, in part, a choice between "home-produced" and "market-produced" housing services. It is interesting because it directs attention to the renovation decisions of housing stock owners.

Housing units can differ in their size, location, age, design, access to other locations, surrounding land uses, associated tax expenditures of local government, etc. There have been two approaches taken to understand this heterogeneity. One approach has households valuing units for their characteristics which yield utility directly or are combined with other inputs, including household time, according to some production function to provide services that determine utility (Becker, 1965; Lancaster, 1966; Muth, 1966). The other approach assumes housing has two dimensions — quantity and quality. This approach is especially useful for analyzing decisions concerning maintenance. In the models of this dissertation, I will focus on issues concerning quality aspects of rental housing and how tenant and owner decisions affect this quality.

The heterogeneity of housing stock precludes the development of an organized commodity market, in the sense of a quoted price for a homogeneous unit. The implication is that accurate price information is not available without a costly search. These search costs are only one component of transactions costs in housing markets. Other components include legal costs, moving costs, costs of readjusting home furnishings to a new unit, and psychic costs. Estimates of these costs range from 8% - 10% of the value of the unit being exchanged. In the first essay of this dissertation, although the heterogeneity of rental units is single-dimensionable as measured by quality, I argue that the market still differs from standard commodity markets. I then model decision-making in the context of asymmetric information concerning tenants' transactions costs.

Many economists have dealt with the related aspect of spatial fixity. I shall not discuss this literature, since it does not contribute significantly toward understanding the models developed in this dissertation.

There is considerable literature, both positive and normative, concerning government involvement in housing markets. With respect to rental markets, the government is involved in two ways: through direct ownership or subsidies and through price controls. Approximately 11% - 12% of the total stock of housing is publicly owned or subsidized by the government and is often rented to tenants at less than cost. The nominal price of housing services is often regulated through rent controls. The effects of these rent controls on the quantity and quality of housing stock have been theoretically modelled and rigorously tested. For instance, quality may suffer as rent controls induce landlords to reduce maintenance expenditures. In the first model I develop, I show that, even without price controls, quality may be inadequately supplied by landlords due to a particular informational problem in rental housing markets.

I will discuss the literature concerning housing stock in existing buildings because theoretical considerations involving the quantity, and, especially, the quality of an existing building over time are implicitly important in the models presented in this dissertation. Of major concern is the lack of consideration of tenant characteristics and choices in determining the quality of a unit in either a static or dynamic context. In a world of imperfect information, the characteristics and decisions of tenants are especially important in modelling rental housing markets, because their choices may not be contractible. Landlord behavior is, of course, still important, but should in some sense depend on their perceptions of how their choices affect the decisions of tenants. Similarly, tenant choices cannot be modelled without consideration of their conjectures concerning how their behavior will affect landlord decisions. This means that quality, in a static or dynamic world, cannot be modelled without explicitly recognizing the strategic nature of choices. That is, it must be examined in a game theoretic context. In this dissertation, I will focus on the informational problems which lead to adverse selection and moral hazard in the context of games. An introduction to adverse selection and moral hazard is reserved for Section 1.3.

In housing markets, maintenance is usually used to describe an activity, taken by the landlord, that augments the quality of a unit of housing. Repainting and repairing, for instance, may be considered maintenance. Renovation is also an activity that augments the quality of a unit, but at a much larger scale – for example, adding a room or redoing a kitchen. In this paper, I will partly focus on maintenance, which can apparently slow or halt the deterioration of quality. It may even increase the quality of an existing dwelling unit.

Most of the existing literature which examines the owner's (not necessarily landlord's) maintenance decisions focuses on the conditions under which a unit will be allowed to deteriorate. Many factors which determine whether a unit will deteriorate, hold steady, or improve in quality have been examined. It may depend on the exogenous rate of depreciation, the maintenance technology, the price of housing services and the price of maintenance inputs over time. For example, Margolis (1981) finds that with constant prices, a constant rate of depreciation, an increasing marginal cost of maintenance, and owners maximizing the present value of their property given the price of housing and initial quality, maintenance will be chosen such that the unit will converge to some constant level of quality. However, under other conditions, the quality may decrease (or even increase). For example, if there is an increasing underlying rate of depreciation, units may decline in quality until they are finally demolished.

Another line of research examines the process of filtering, in which a dwelling unit declines in quality, changes in relative price, and is occupied by lower-income households. Related to this, recent attention has been given to the process of gentrification, in which older units are renovated and lower-income households are replaced by higher-income households.

Although some economists have incorporated demand-side as well as supply-side characteristics in modelling owner maintenance decisions, the focus in all of the above models has been only on owner decisions concerning maintenance. Little attention has been given to the decisions of the tenant which can affect quality in the context of a rental housing model. Henderson and Ionnides (1983) emphasized a problem, which they call a rental externality, where if a unit is rented to tenants excessive deterioration will occur relative to if it were owner occupied. Vorst (1987) implicitly incorporates tenant choices which can affect quality into their model of landlord maintenance by assuming that one component of a stochastic influence on quality is associated with tenant behavior. Guasch and Marshall (1987) note that the size of tenure discounts may depend on whether a tenant reveals himself to be good or bad, in the sense of maintaining a unit. Kanemoto (1990) recognized the importance of tenant behavior in affecting landlord investment decisions and the nature of efficient contracts in housing markets. Although these models recognize to some degree the importance of tenant behavior, they do not study tenant behavior explicitly. Instead, they focus on owner decisions in the context of tenant decisions which may affect quality.

In this dissertation, I will examine three different models where decisions of the tenant are important factors influencing equilibrium quality. In these models, it is recognized that a source of inefficiency with respect to rental housing markets may involve some informational asymmetry. In particular, the tenant and/or landlord may possess certain information or take certain actions which may not be observable to the other or verifiable by the courts. In general, these informational problems lead to models of adverse selection or moral hazard. In the next section, I shall provide a brief introduction to the problems of adverse selection and moral hazard.

However, before proceeding to the next section, I will discuss the rental housing literature concerned with the market adjustment process. This literature is, to some degree, related to certain aspects of this research. In particular, it has been observed that rental housing markets may be characterized by excess demand or excess supply at the market price. Note that this idea concerns market adjustment toward some "natural" vacancy rate greater than zero. This is because vacancies are a natural part of any market characterized by search and turnover. So excess demand does not imply zero vacancies. Early work by Blank and Winnick (1953) suggests that landlords are slow to respond to an increase in demand by raising rents. They may simply accept increases in income resulting solely from the increase in occupancy. Thus, vacancies may fall below the natural rate.

The idea of nonmarket-clearing is interesting because many markets characterized by informational problems of the type developed in this dissertation also exhibit nonmarket-clearing. This indicates that there is some analogy, which has yet to be exploited, between these markets. One seemingly fruitful avenue for research would exploit labor market search theory to explain nonmarket-clearing prices in rental housing markets. Although I will not develop the search analogy further, the ideas introduced below should offer some additional insight into the connection between rental housing markets and other markets, including the labor market.

1.3: Introduction to Models of Adverse Section and Moral Hazard

The two categories of models relevant for this dissertation are models of adverse selection and moral hazard. Both types involve what is called asymmetric information, where some player has useful private information. These types of models are not the only ones which involve asymmetric information. Included in the category of asymmetric information are models of signalling and screening. Also, some separate moral hazard into games of hidden actions and games of hidden information. In this section, I will not discuss signalling, screening, or moral hazard with hidden information. For a clear description of what is similar and distinct between these models, see Rasmusen (1989, pp. 133-136).

I shall employ the principal-agent framework in introducing both adverse selection and moral hazard. In these models, used to analyze asymmetric information, there are two (sometimes representative) players, a principal and an agent. The principal hires an agent to perform a task and the agent either has or somehow obtains some private information. In the case of adverse selection, this information concerns the "type" of agent he is. In the case of moral hazard (with hidden actions) it concerns his actions. The key in each of these models is the inability of the principal to verify, or even observe or infer, the "type" of the agent or the action taken by the agent.

In a typical adverse selection model, nature takes an initial move to determine a "type" for the agent. The agent knows his type, but the principal does not. The principal offers a set of contracts to the agent. The agent then accepts at most one contract from this set. The payoffs to the principal and agent are then determined by the contract and type of agent. A typical result is that equilibrium is inefficient. Also, if we extend the model to include competition, equilibrium may not exist.

The first example comes from Akerlof (1970). To demonstrate his model, applied in the context of the used car market, suppose nature has chosen half the cars in the market to be of quality q^G, good cars, and the other half of lower quality q^B, lemons, where $q^G > q^B$. If the buyer, the principal, and the seller, the agent, are risk neutral and quality is valued at one dollar per unit, after a trade, the buyer's payoff is $q - p$ and the seller's payoff is $p - q$, where $q \in \{q^B, q^G\}$ and p is the price.

If the buyer could observe q at the time of purchase, he would accept a contract to pay q^G for a good car and q^B for a bad car, a lemon. However, q is not contractible for some reason. If the seller offers $p' = .5q^B + .5q^G$, a price equal to average quality, the buyer would know that only cars with quality q^B will be sold. Thus, the buyer would refuse to pay more than q^B. If the seller sells at q^B, then only half the cars will be traded and they will all be lemons. If we allow the quality types to be uniformly distributed between q^B and q^G, the market for used cars will completely disappear. In each of these cases, trade diminishes as a result of the adverse selection problem, but because the buyer and seller value cars identically, there is no inefficiency. If we assume buyers value cars more than sellers or assume sellers' valuations differ we get diminished trade and inefficiency. Other extensions of Akerlof's basic model yield similar results. However, Kim (1985) constructs a model which retains the basic informational structure consistent with adverse selection, yet obtains efficient market clearing. His model relies on endogenously determined quality (through maintenance) and two periods. Thus, it is not merely a simple extension of the basic model.

Rothschild and Stiglitz (1976) develop a model of competitive insurance markets, where consumers know their accident probabilities, while insurance companies do not. Absent an accident, the individual has income W. With an accident, his income becomes $W - d$. However, by buying insurance with premium α_1 and payment α_2', should an accident occur, his income becomes

$W_1 = W - \alpha_1$, absent an accident, and $W_2 = W - d + \alpha_2$, where $\alpha_2 = \alpha_2' - \alpha_1$, with an accident.

First, note that, when there is no heterogeneity between customers, equilibrium, characterized by maximizing customer expected utility subject to zero expected profit for the company, will occur with full insurance. That is, $W_1 = W_2$ or $d = \alpha_2'$.

Now suppose that there are two types of customers: low-risk individuals with accident probability p^L and high-risk individuals with accident probability p^H. If λ is the fraction of high-risk individuals, a single contract will have an average accident probability of $p^A = \lambda p^H + (1 - \lambda)p^L$. In the competitive case, any single contract yielding zero profit for the insurance company cannot be a Nash equilibrium. The reason is that, given this contract, there will always exist an alternative contract offer, preferred only by low-risk individuals, which is profitable to offer. If any equilibrium exists, it must separate high-risk individuals from low-risk individuals. However, when it is possible to separate individuals according to risk by offering multiple contracts, Rothschild and Stiglitz show that high-risk customers are fully insured, while low-risk customers are not.

The above mentioned possibility of no Nash equilibrium led to the consideration of alternative concepts of equilibrium. One example is due to Wilson (1977). In his definition, companies offering new policies conjecture the withdrawal of contracts which become unprofitable due to their contract offer. Using this alternative definition, existence of equilibrium can be established. In Chapter 2, I shall employ both the Nash definition and Wilson definition in characterizing adverse selection in rental housing markets.

Another application of adverse selection developed to explain the observed excess demand for credit at the market interest rate. One such model was developed by Jaffee and Russell (1976). The idea is that banks can improve the pool of borrowers (decrease the probability of default) by reducing the amount loaned to individuals. In Chapter 2, I build on their model to examine the implications of adverse selection in rental housing markets. In Stiglitz and Weiss (1981), the pool of loan applicants and their behavior changes, because of adverse selection and moral hazard. The adverse selection is that borrowers who are likely to default are not as concerned about the high interest rate as those who are not likely to default. The result is that the number of loans forthcoming in equilibrium will shrink and the default level will rise. The moral hazard is that borrowers shift to higher-risk, greater-expected-return projects as the interest rate rises.

A few additional notes concerning adverse selection models are in order. The lemons problem introduced by Akerlof (1970) is merely a special case of a broader dependence of quality on price. For a survey of these ideas, see Stiglitz (1987). I have not discussed all the models which have employed the idea of adverse selection. For instance, Bagehot (1971) examines an adverse selection explanation for the bid-ask spread in options markets. Guasch and Weiss (1980) examined adverse selection and the willingness of workers with different abilities to take tests. There are many examples like these which have no direct relevance to the models introduced in this dissertation and, thus, are not discussed.

I should also note some of the many ideas concerning possible solutions to the problem of adverse selection. For example, in the lemons market, one possibility would simply make the quality contractible through perhaps, government imposed quality standards or private warranty contracts policed by the courts. The idea here is quite similar to certification and occupational licensing in markets for professionals. Reputation might also solve adverse selection. This is similar to the idea that repeat purchases provide incentives for firms to maintain quality, an idea dealt with primarily in the context of moral hazard. However, notice that this solution requires the introduction of a dynamic model. At least as far as this dissertation is concerned, this idea will not be pursued in the context of adverse selection.

Next, I will discuss some simple static models of moral hazard. I shall reserve discussion of more complex models of double moral hazard and dynamic games for Section 3.2 of this dissertation. In the simplest, principal-agent set up of moral hazard, the principal and agent first agree to a contract, but then the agent takes an action unobserved by the principal. Of course, the payoffs to the principal and agent depend on the action taken by the agent. The problem is that, because the action taken by the agent is not contractible, it will often be inefficient.

There are seemingly endless applications stemming from the basic model. One formulation would have a manager as the principal and worker as the agent. Related to this, stockholders can be the principal and the manager the agent. For examples of how these models have found their way into the management literature, see Milgrom and Roberts (1992) and Rubin (1990). In insurance markets, policyholders are agents for the insurance company, where the actions are either the care taken to avoid theft, fire, or adverse health. The sharecropper is an agent for the plantation owner. Doctors and lawyers are agents for patients and clients. This list does not exhaust application in the existing literature, while the list of applications continues to grow. I shall expand this list by examining moral hazard in rental housing markets in Chapter 3 and Chapter 4 of this dissertation. The tenant is an agent for the landlord and the landlord is an agent for the tenant. This is an example of double moral hazard. Again, discussion of these models is reserved for Section 3.2.

Consider a simple model where a manager offers a wage function, w(q), where q is output. The worker first decides whether or not to accept the contract. If he accepts the contract, he then chooses effort, e, unobservable to the manager. The output, q, is a function of effort. That is, $q = q(e)$. This output is observable to both the manager and worker in the simplest formulation. Assuming the manager gets all the gains from trade, he wishes to make the agent choose e^*, which gives the efficient output q^*. Let $U(e,w(q))$ be the worker's utility function, with $U_e < 0$ and $U_w > 0$. The contract must satisfy the participation constraint for the worker with equality. That is, $U(e^*, w(q^*)) = \bar{U}$, the reservation utility. The contract can then be set to yield strictly lower utility for $q(e) \neq q^*$, or $U(e, w(q)) < \bar{U}$, for $e \neq e^*$. Any such contract will induce the worker to choose $e = e^*$. One example would be to offer $w = w^*$, for $q = q^*$, where $U(e^*,w^*) = \bar{U}$, and $w = 0$, for $q \neq q^*$. Contracts like these are called "forcing" contracts. Notice that efficiency obtains despite the inability of the manager to directly observe e. This is because q is observable and the manager knows the function q(e). The manager can infer effort from quantity.

Now suppose that, unobserved by the principal, nature chooses $\theta \in \Re$, according to the probability density function $f(\theta)$, and $q(e,\theta)$. In this case, assuming differences in risk preferences, the outcome is inefficient. The problem is that effort can no longer be inferred from output. Output may be low due to the lack of effort or due to nature. The manager wants to provide incentives, but also reduce the risk assumed by risk-averse workers. Increasing incentives by making the payment to the agent more responsive to output generally increases the expense of providing for agent participation, because it increases the risk to the worker. So the manager must tradeoff incentives and risk reduction, settling for inefficient output in order to provide insurance consistent with worker participation.

Notice that in models of competitive insurance markets, where the action taken by the insured is not contractible, we get results similar to those in adverse selection models. In particular, equilibrium with full insurance will not obtain. Equilibrium will entail deductibles or coinsurance.

Again, this introduction is not meant to exhaust the literature on moral hazard. In fact, that is not the purpose of this dissertation. The variety of applications mentioned earlier should suffice; however, to illustrate that moral hazard is a common problem.

Of course, the literature concerning adverse selection and moral hazard is much richer than it may seem from the above introduction. All I have attempted to do is highlight the problems, without offering much insight into possible solutions. My intention is to provide a framework useful to understand the relationships within and between the following three chapters.

1.4: Introduction to the Models of Chapters 2 - 4

In this section, I will provide an introduction to the original models of rental housing markets presented in the following chapters. As I mentioned above, they all have asymmetric information in common. Also, they all incorporate the important choices of tenants in influencing landlord behavior and equilibrium. The model of Chapter 2, however, differs substantially from those of Chapter 3 and Chapter 4. In particular, the model of Chapter 2 is developed in the context of adverse selection and is static. The models of Chapter 3 and Chapter 4 are developed in the context of moral hazard and are dynamic. However, although both Chapter 3 and Chapter 4 present dynamic models with moral hazard, they are also different. For instance, in Chapter 3, the relationship between a particular tenant and landlord may continue indefinitely, while in Chapter 4, the relationship lasts only one period. The reason for the distinction is that in Chapter 4, I wish to explicitly consider the role of damage deposits, while there may be no such role in Chapter 3.

Again, the unifying theme of these models, beyond that they all obviously deal with the rental housing market, is that they all involve asymmetric information. The results in these models are not unlike the standard inefficiency results obtained in other markets characterized by asymmetric information. However, as I discussed in Section 1.2, there are many unusual characteristics of rental housing markets. These characteristics provide the basis which differentiates these models from those in the existing literature. The simple observation that these ideas have not been fully developed in the context of rental housing markets provides support for their originality and, to some extent, their complexity.

In Chapter 2, tenants possess private information about their transactions costs of moving. The result, as we shall see, is that when Nash equilibrium obtains, it must separate tenants who are good, in that they exert effort to maintain quality, from those who are bad, in that they exert no effort to maintain quality. In fact, in this model, good tenants are always worse off, in equilibrium, as a result of this informational problem than they would be in a first-best perfect information world. The existence of Nash equilibrium is then discussed.

In Chapter 3, the asymmetric information problem takes the form of moral hazard. The tenant is an agent for the landlord and the landlord is an agent for the tenant, when the relationship continues over time. If the relationship is allowed to continue indefinitely, I show that there exists a very simple rental contract, contingent upon the history of quality, which supports Pareto efficiency in a subgame perfect equilibrium. This is accomplished through the use of strategies not independent of the history of quality and, hence, the past choices of both the tenant and landlord.

In Chapter 4, the relationship is assumed to last only one period. With only tenant moral hazard, I show that damage deposits develop to preserve efficiency in equilibrium. When extended to double moral hazard, however, as expected in light of Chapter 3, Pareto efficiency cannot obtain. However, under certain assumptions regarding the relative cost of effort versus maintenance, it is established that damage deposits can play a positive and effective role in supporting a more efficient equilibrium than would otherwise occur.

CHAPTER 2
QUALITY RATIONING IN RENTAL HOUSING MARKETS

2.1: Introduction

There are many peculiarities in the rental housing market which make formal microeconomic modelling difficult. These peculiarities largely develop from the characteristics of the commodity itself. In the introductory chapter of this dissertation, I discussed some of these characteristics. For a further introduction to the modelling responses to the resulting difficulties see Stahl (1985) and Smith, Rosen, and Fallis (1988). I shall abstract from many of these characteristics in order to analyze a particular informational asymmetry problem. Building on a model by Jaffee and Russell (1976) to explain credit rationing, I will demonstrate that a type of quality rationing may develop in a market characterized by landlord uncertainty regarding the type of tenants who will occupy his units. Throughout this chapter, quality rationing is said to occur when landlords quote an implicit price per unit of quality then proceed to offer a lower level of quality than that demanded by their tenants. As in Jaffee and Russell (1976) this type of rationing can be explained on a fundamental level by employing the same principles of moral hazard and adverse selection which have been used to explain a variety of market failures. These models are by now well known and have found their way into many surveys and textbooks. (See, for example, Phlips,1988, and Rasmusen, 1989.)

In modelling the rental housing market to illustrate this quality rationing, I shall concentrate upon two unique aspects of the rental housing market. The first is concerned with the nature of the rental housing market itself and the second with an associated informational problem.

The rental housing market does not work the same as textbook competitive markets. A landlord will offer a unit of some quality level for some rental rate. In effect, he is offering a specified quality level at some imputed price per unit of quality. Also, a tenant consumes at most one dwelling unit and thus, one price and quality combination. The tenant does not purchase as much quality as he wishes at the imputed price, but rather consumes only one dwelling at the quoted rental rate. This is in contrast to a market where consumers purchase as much of a product as they are willing and able to purchase at a given per unit price.

Rothschild and Stiglitz (1976) note a similar characteristic in their model of insurance markets. Under price competition, insurance firms set a price and allow consumers to buy as much or as little insurance as they want at that price. However, in what they call price and quantity competition, companies offer sets of contracts, with each contract specifying a price and quantity, and consumers may buy at most one contract. In fact, they go further and argue that, in the context of their model, price competition is a special case of price and quantity competition.

Another characteristic of rental housing markets is that rental contracts are realistically incompletely specified. Therefore the supplier of rental housing is confronted with the risk of transferring some of his property rights to a tenant with characteristics such as sociability, housekeeping manners, or payment morale, that are unknown to him ex ante. Price setting then depends on the landlord's evaluation of the renters characteristics relevant to him, and on the distribution of these characteristics among tenants active in the market.

In this chapter, I incorporate these unique characteristics in a model of rental housing to show how quality rationing develops as a market response to adverse selection. There are two types of agents, landlords and tenants. The tenants differ only in terms of their transactions costs of moving, which ultimately divides them into two groups: "good tenants" and "bad tenants." Good tenants provide the effort necessary to maintain the agreed upon quality level while bad tenants do not. Landlords, however, are unable to distinguish between individuals according to these "moving costs." The focus of this chapter is to analyze landlord and tenant behavior in a context where a continuum of types of tenants are known to exist, but in which individuals can be identified only ex post by their actual behavior.

Throughout the discussion of market behavior, I will employ the Nash definition of equilibrium. In this definition, landlords conjecture no response from their competitors when offering a rental contract. As I will show, when a Nash equilibrium exists, there is quality rationing to good tenants. However, I will also show that a Nash pooling equilibrium does not exist. Thus, in equilibrium, good tenants are separated from bad tenants.

In Section 2.2, the basics of the model are presented. Tenant behavior and their implications for the proportion of tenants who choose expend effort to maintain quality are discussed in Section 2.3. In Section 2.4, landlord behavior and market equilibrium are defined and analyzed. The main result that, in equilibrium, quality is always rationed to good tenants is established in Section 2.4. The existence of Nash equilibrium is considered in Section 2.5. In Section 2.6, some concluding remarks are offered.

2.2: The Model

The setting for this model is a regional rental housing market. As usual in rental housing market models, dwellings are classified according to certain characteristics such as floor space, number of rooms, and year of construction, thus yielding different "types" of dwellings. In this model all units under consideration are of the same type and the type of dwelling does not change with respect to the above characteristics over the time period considered. However, the quality of certain components of the unit can be controlled by the landlord. These components, the quality of which must be determined at the beginning of the period, include the carpet, drapes, interior walls, certain mechanical fixtures, etc. The quality of these components can be aggregated into a single measure denoted by q ($q \geq 0$). Higher values of q imply higher levels of quality. The variable q is continuous and all agents have identical preferences over q with, of course, higher quality preferred to lower quality.

From the point of view of the landlord, there will be two types of tenants, good and bad, which will be endogenously determined in the model. Whether a tenant is good or bad in equilibrium will, as we shall see, depend on that tenant's transactions costs of moving, which are not known a priori to the landlord.

A landlord will offer a unit of some quality level q for some rental rate. In effect, he offers a particular amount of quality at some (imputed) price per unit of quality. A rental contract is a two-dimensional vector $s=(p,q)$, where p is the imputed price per unit of quality and q is quality. The space of rental contracts is

$$\overline{S} = \{ s \in R^2 \mid p \geq 0, q \geq 0 \}.$$

Keep in mind that the quality and quantity of all other characteristics of the rental unit not measured by q are fixed. The results of this chapter deal with this implicit market for quality.

2.3: Tenant Behavior

All tenants are identical with respect to their utility functions. Let $U(x,q) = x + U(q)$ be the utility function, where x is a composite commodity other than the dwelling unit in question and q is the quality of the dwelling unit. This restriction regarding tenant utility simplifies our analysis, but does not affect our main result that quality is rationed to good tenants, in equilibrium. Assume $U'(q) > 0$ and $U''(q) < 0$.

Although tenants are identical in all other respects, they do have different "transactions costs." These costs must be incurred by the tenant in the event he leaves the unit and can be associated with searching and moving. Search costs are usually the direct result of the heterogeneity and spatial dispersion of units. Although I abstract from this heterogeneity, information concerning vacancies may be costly to obtain. Other components of transactions costs may include the costs of contracting and the costs associated with moving. That these costs differ across tenants can be attributed to differences in search ability, quantity and bulk of furniture, psychic costs, etc. It is these differences in transactions costs which will ultimately determine whether a tenant is a good tenant or a bad tenant. This will be explained in more detail below as the budget constraint and utility maximization are discussed.

The particular values of the above transactions costs will be the source of the informational asymmetry. Of course, each tenant knows his own transactions costs; however, the landlords cannot distinguish between tenants according to these costs. Landlords know the distribution of transactions costs.

The budget constraint for a tenant who chooses a dwelling of quality q and chooses to maintain that quality is given by

$$p\,q + x + e(q) = I, \tag{2.1}$$

where p is the imputed price per unit of quality and I is exogenous income. The price of x is assumed to be equal to unity. The continuous effort function, e(q), measures (in terms of x) the amount of effort required to maintain a unit of quality q so that quality is reduced by only a fraction δ. The fraction δ reflects only "normal" wear on the components which provide quality. This effort function can be thought of as the sacrifice which must be made in order that only normal wear on the quality components take place. Assume $e(0) = 0$, $e'(q) > 0$, and $e''(q) \geq 0$. That is, the higher the quality the greater the effort necessary to prevent excessive wear. Also, this effort increases at a nondecreasing rate as quality increases.

Rearranging (2.1) and substituting into the utility function yields the unconstrained maximization problem

$$\max_{q} \, [\, I - p\,q - e(q) + U(q)].$$

The first-order condition for this solution is

$$\frac{dU}{dq} = -\,(\,p + e'(q)) + U'(q) = 0. \tag{2.2}$$

From this we can write the demand for quality function $q^* = q_G{}^*(p)$. The subscript "G" is useful because I am, for now, assuming the tenant performs effort $e(q^*)$. We know $\frac{dq^*}{dp} = \frac{1}{U''(q) - e''(q)} < 0$. Assume $q^* = 0$ at some finite p.

It is useful at this point to derive the shape of iso-utility curves in price-quality space. To obtain information about the shape of a good tenant's iso-utility curve, differentiate

$$I - p\,q - e(q) + U(q) = k \tag{2.3}$$

with respect to p and q and solve to obtain

$$\frac{dp}{dq} = \frac{U'(q) - [p + e'(q)]}{q} \tag{2.4}$$

From (2.2), the slope of this iso-utility function will be zero if and only if the point lies on the demand function. Further information about the shape of the iso-utility function can be obtained by differentiating (2.4) with respect to q and arranging terms. This gives

$$(2.5) \qquad \frac{d^2p}{dq^2} = \frac{U''(q) - 2\frac{dp}{dq} - e''}{q}.$$

The denominator in (2.5) is positive. The first term in the numerator is negative. The last term in the numerator (without sign) is nonnegative. The slope of the iso-utility function is zero where it intersects the demand curve. Thus, in the neighborhood of the demand function the iso-utility function is concave. The slope of the function can change sign only at the point of intersection with the demand curve: if it did otherwise, then there would be a point with zero first derivative and a positive second derivative, and this is ruled out above. Therefore, the iso-utility curves for good tenants are monotonically rising until they reach the demand function and monotonically falling thereafter. Figure 2.1 illustrates the demand function, $q_G{}^*(p)$, and representative iso-utility curves. In figure 2.1, $U^i_G(s)$, where i =1,2,3, represent different levels of utility with, higher utility obtained as the tenant moves to the southeast along the demand curve.

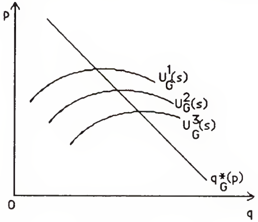


Figure 2.1. Good tenant iso-utility curves

The above discussion assumes that the tenant will find it optimal to expend e(q) to maintain the unit regardless of quality. This will not, however, be the case for all tenants. In the analysis which follows, a tenant who expends e(q) at the relevant q will be referred to as a good tenant. A tenant who sets $e(q) = 0$ at the relevant q will be referred to as a bad tenant. The type of tenant an individual becomes is determined by his particular transactions costs. These transactions costs are also measured in terms of x and are denoted by T. The particular T value of a tenant is not known by the landlord ex ante. Because tenant effort can only affect the end-of-period quality, the type of tenant who occupies a unit only becomes known to the landlord at the end of the period. If at the end of the period the tenant reveals himself to have been good, the landlord allows that tenant to remain in the unit and no transactions cost is incurred. Alternatively, if the tenant is revealed to be bad, he is forced to leave the unit and incurs his transactions cost T. I will make the assumption that eviction is costless for the landlord and, in this static model, the ability to rerent the unit is irrelevant. Since each tenant wants to maximize his utility, he will be a bad tenant and expend zero effort to maintain quality q if $e(q) > T$. Assume for simplicity, that the damage above normal wear which occurs at each level of quality is the same for each bad tenant.

For some $q = \hat{q}$ a tenant will maximize utility by following either a good course or a bad course. In particular, if $e(\hat{q}) = T$, then $I - p\hat{q} - e(\hat{q}) + U(\hat{q}) = I - p\hat{q} - T + U(\hat{q})$. A tenant will thus choose to be bad if $T < e(\hat{q})$. That is, whenever the transactions cost for a tenant is less than the effort necessary to maintain $\hat{q}$, tenants choose the bad course by expending zero effort.

Now allow T to vary among the tenants. Assume that the distribution of T across tenants is continuous with support $[T_{min}, T_{max}]$. Define q_{min} by the equation $T_{min} = e(q_{min})$. Assume T_{min} is bounded above zero so that q_{min} is also bounded above zero. From these assumptions we can determine a function $\lambda[q]$, where λ is the proportion of tenants who will expend e(q) to maintain a unit with quality level q. That is, λ is the proportion of good tenants at q. Properties of λ are

(2.6) $\lambda[q] = 1$ for $q \leq q_{min}$ and

(2.7) $0 < \lambda[q] < 1$ and $\lambda'[q] < 0$ for $q > q_{min}$.

These equations show that there will be only good tenants if $q \leq q_{min}$ and the proportion of good tenants declines as q increases beyond q_{min}. The initial strict inequality in (2.7), $0 < \lambda[q]$, depends on at least one tenant always being a good tenant (i.e. T_{max} is sufficiently large). Figure 2.2 shows $\lambda[q]$.

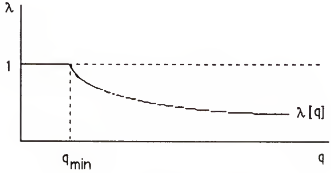


Figure 2.2. $\lambda[q]$.

For $q > q_{min}$ there will always exist some bad tenants. By examining the utility maximization problem of a tenant who is bad even at very low levels of quality, we can derive a demand function and iso-utility function with similar properties as those for a good tenant. The bad tenant maximizes $I - pq - T + U(q)$ with respect to q. This gives the first-order condition

$$\frac{dU}{dq} = -p + U'(q) = 0. \tag{2.8}$$

This defines a quality demand function for bad tenants, $q_B^*(p)$. The slope of the iso-utility function is

$$\frac{dp}{dq} = \frac{U'(q) - p}{q} \tag{2.9}$$

and we can obtain

$$\frac{d^2p}{dq^2} = \frac{U''(q) - 2\frac{dp}{dq}}{q}. \tag{2.10}$$

By using a proof similar to that for good tenants, it can be shown that the bad tenants have iso-utility curves with properties similar to those of good tenants.

Lemma 2.1. (Single Crossing Property) An iso-utility curve for a bad tenant may intersect an iso-utility curve for a good tenant only once.

Proof. (By contradiction) Suppose there is more than one intersection. Then the iso-utility curve for a bad tenant must cross the good tenants iso-utility curve at least once from below and at least once from above as q increases. We know

$$\frac{dp}{dq} = \frac{U'(q) - [p + e'(q)]}{q} \quad \text{and} \tag{2.4}$$

$$\frac{dp}{dq} = \frac{U'(q) - p}{q}. \tag{2.9}$$

For any contract in $\bar{S}$ (2.9) is greater than (2.4). By examining the absolute values of each slope, it is clear that the bad tenant iso-utility curve must always intersect the good tenant iso-utility curve from below as we move to higher levels of quality. Since iso-utility curves, within each family of iso-utility curves, do not cross each other, we have a contradiction, and there cannot be more than one intersection by these two iso-utility curves. Q.E.D.

2.4: Landlord Behavior and Market Equilibrium Under Competitive Conditions

Assume that the cost of providing a quality level is given by $C(q) = (r + \delta)q$ with $r, \delta > 0$, where r is the competitive rate of return and δ is the "normal care" quality depreciation rate. Note that I am measuring quality produced in dollars. If a tenant does not maintain and care for the unit, the landlord incurs an additional cost at the end of the period. Assume that this additional cost equals mq, where $m > 0$. Also, assume $mq > e(q)$ for all q, or else it would be suboptimal to have tenants expend effort to maintain quality and the problem becomes uninteresting. This additional cost mq is that which is required to bring quality back to the level which reflects only normal wear.

All landlords have identical costs. They maximize the expected value of their profit over price and quality. Their expected profits can be written as

(2.11) $E\pi = pq - (r + \delta)q - (1 - \lambda[q])mq.$

Remember that λ is the the proportion of good tenants. If $\lambda = 1$, $E\pi = pq - (r + \delta)q$. If $\lambda = 0$, $E\pi = pq - (r + \delta + m)q$. In general, the proportion of good tenants at any contract will depend not only on e(q) and the distribution of T, but also on what other contracts are offered. Equation (2.11) will be most useful in describing expected profits when all landlords offer the same single contract set.

Before continuing, it must be made clear what characterizes equilibrium in this competitive market. Generally, a set of contracts is an equilibrium if, when that set is offered, no landlord has an incentive to change his offer. In the Nash definition below, I assume that each landlord conjectures that the offers of the other landlords will not change in response to his own contractual variation.

Definition 2.1. (Nash) A set of contracts is an equilibrium if

(i) each contract in the set earns nonnegative profits and

(ii) there is no other set of contracts which, when offered in addition to that set, earn strictly positive profits for each contract in the set.

It is important to note that, in this chapter, a "profitable" contract is one which makes strictly positive profits. When considering a contract offer, I will sometimes refer to a contract which just breaks even as "unprofitable" to offer. This is to make it clear that for an alternative set of contracts to break a potential Nash equilibrium, all contracts in the set must earn strictly positive profits or else it is not "profitable" to offer them.

The following two Lemmas will be useful in deriving the main results of this chapter.

Lemma 2.2. (Zero-Profit Condition) If a Nash equilibrium exists each contract in that set must earn zero profits.

Proof. To prove this, I will show that any set of contracts with at least one contract earning strictly positive profits or negative profits cannot be a Nash equilibrium. Suppose there exists a set of contracts $\tilde{S} \subset \bar{S}$ where at least one contract in the set earns strictly positive profits. Choose any one of these contracts and denote it by contract $\tilde{s} = (\tilde{p},\tilde{q})$. Since costs depends only on q, with continuity there will exist another single contract $\tilde{s}' = (p',\tilde{q})$ where $p' < \tilde{p}$ and profit is strictly positive. Thus, $\tilde{s}'$ breaks $\tilde{s}$ as a Nash equilibrium.

Also, if there exists a contract set with at least one contract earning strictly negative profits, it cannot be a Nash equilibrium since it violates the nonnegative profit condition. Q.E.D.

Lemma 2.3. A set of contracts is a Nash equilibrium if and only if there does not exist a single, strictly profitable, contract which could be offered, given that set.

Proof. In this proof, I will show that if a multiple contract set of contracts (consists of two or more contracts) breaks a candidate Nash equilibrium, then one contract in that set, if offered by itself, would break the candidate Nash equilibrium. This means that if no such single contract offer exists, no multiple contract offer can break the candidate Nash equilibrium. That is, a necessary condition for a multiple contract set to break a candidate equilibrium is that there exists a single contract offer which by itself can break the candidate Nash equilibrium.

Assume $\hat{S}$ in $\bar{S}$, is a multiple contract set of contracts which can break S^N in $\bar{S}$ as a candidate Nash equilibrium. By definition, each contract in $\hat{S}$ must earn strictly positive profits when offered, given S^N. There are six possibilities regarding the types (good or bad) of tenants contracts in $\hat{S}$ can attract, given S^N. They can attract (1) only good tenants, (2) only bad tenants, (3) good tenants and bad tenants at each contract in $\hat{S}$, (4) only good tenants at contracts in S^G, a subset of $\hat{S}$, and only bad tenants at contracts in $S^B = \hat{S} - S^G$, (5) only good tenants at contracts in S^G, a subset of $\hat{S}$, and good and bad tenants at contracts in $S^{GB} = \hat{S} - S^G$, and (6) only bad tenants at contracts in S^B, a subset of $\hat{S}$, and good and bad tenants at $S^{BG} = \hat{S} - S^B$. I shall consider each of these cases separately and the definitions regarding the subsets of $\hat{S}$ are not necessarily the same between cases.

In case (1), all good tenants who choose contracts in $\hat{S}$ must be indifferent between each contract in $\hat{S}$. Hence, offering just one contract in $\hat{S}$ would attract all of these good tenants. Thus, this contract, when offered by itself, would be profitable to offer, given S^N.

In case (2), all bad tenants who choose contracts in $\hat{S}$ must be indifferent between all contracts in $\hat{S}$. Hence, offering just one contract in $\hat{S}$ would attract all of these bad tenants. Thus, when this contract is offered by itself, if would be profitable to offer, given S^N.

In case (3), good tenants and bad tenants must be indifferent between every contract in $\hat{S}$. But this is impossible, by the single-crossing property. Only one contract can lie on both a good tenant's iso-utility curve and bad tenant's iso-utility curve. Any set of contracts which makes good tenants indifferent cannot also make bad tenants indifferent.

In case (4), any contract in S^B would, at worst, attract just bad tenants. Thus, it continues to be profitable to offer by itself, given S^N.

In case (5), there can be only one contract in S^{GB}, by the single-crossing property. Good tenants must be indifferent between this contract and all contracts in S^G. If only the contract which attracts both good and bad tenants is offered, it can only become more profitable. This is because only more good tenants will be attracted to it.

In case (6), by the single-crossing property, there can be only one contract in S^{BG}. Bad tenants must be indifferent between this contract and all contracts in S^B. If only one contract in S^B is offered, it, at worst, attracts only bad tenants. Thus, it continues to be profitable to offer, given S^N. Q.E.D.

It will be useful to examine sets of contracts which contain only a single element. In this case, Lemma 2.2 implies

$$p = (r + \delta) + (1 - \lambda[q])m. \tag{2.12}$$

Equation (2.12) defines the set of single-contract sets which satisfy the zero-profit or "breakeven" constraint. Hereafter, I refer to the right-hand side of (2.12) as the average cost function, $AC^*(q)$.

Figure 2.3 illustrates the properties of $AC^*(q)$. For $q \le q_{min}$, $AC^*(q) = r + \delta$, since $\lambda[q] = 1$. Beyond q_{min}, the average cost function is positively sloped. This can be shown by taking the derivative of average cost with respect to q to get

$$(2.13) \qquad \frac{dAC^*(q)}{dq} = - \lambda'[q]m > 0.$$

The inequality follows since $\lambda'[q] < 0$ for $q > q_{min}$ and $m > 0$. Note that $AC^*(q)$ approaches $r + \delta + m$ asymptotically if $T_{max} = \infty$. In this chapter, I consider only the interesting case where $q^* = q_G^*(p)$ intersects $AC^*(q)$ above q_{min}. If this were not the case, then the results concerning quality rationing will not hold. However, note that this is not a very restrictive assumption since it is likely that T_{min} will be sufficiently low so that $q_G^*(p)$ intersects $AC^*(q)$ above q_{min}.

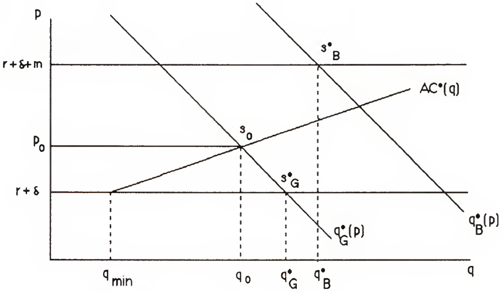


Figure 2.3. $AC^*(q)$.

Zero profits and market-clearing would imply that contract $s_o = (p_o,q_o)$, in figure 2.3, is offered by landlords. This contract is standard in most discussions of equilibrium, since at p_o "demand" equals "supply". Note that at s_o, good tenants pay a price greater than the average cost of providing q_o. This is a result of the existence of bad tenants who are indistinguishable from good tenants prior to the end of the period. Notice that if all tenants were good tenants, contract s_G^* would be the single-contract equilibrium. If some tenants are bad and landlords had perfect information about transactions costs, the set (s_G^*, s_B^*) would be the multi-contract "separating" equilibrium. As I will show next, s_0 can never be a Nash equilibrium.

Proposition 2.1. Contract $s_0 = (p_0, q_0)$ cannot be a Nash equilibrium.

Proof. To prove this I will show that there exists another single contract offer which is strictly profitable to offer, given s_0. Let $\tilde{s} = (\tilde{p}, \tilde{q})$ be any contract such that $U_G(\tilde{s}) = U_G(s_0)$ and $\tilde{q} \leq q_0$. In the neighborhood of s_0 it must be true that for any $\tilde{q}$ in this neighborhood $\tilde{p} > AC^*(\tilde{q})$ since the slope of the iso-utility function of good tenants evaluated at s_0 is equal to zero and $\frac{dAC^*(q)}{dq} > 0$. This implies strictly positive profits at $\tilde{s}$. From Lemma 2.2, $\tilde{s}$ cannot be an equilibrium, since there exists another single contract $\tilde{s}' = (p', \tilde{q})$ where $p' < \tilde{p}$ and $p'\tilde{q} - (r + \delta)\tilde{q} - (1 - \lambda[\tilde{q}])m\tilde{q} > 0$. This is because $U_G(p', \tilde{q}) > U_G(\tilde{p}, \tilde{q})$. Since $U_G(\tilde{s}) = U_G(s_0)$ by assumption, $\tilde{s}'$ will also break s_0 as an equilibrium. Q.E.D.

In the proof of Proposition 2.1, I demonstrated that there will always exist a contract a landlord can offer which makes positive profits given s_0. This is true because there will exist a non-empty set of contracts which offer greater utility to good tenants. In figure 2.4, as long as the new offer lies in the shaded area, it will always break potential equilibrium at s_0. Since this set is bounded below by $AC^*(q)$, each contract in the set will be profitable to offer regardless of whether or not bad tenants also prefer contracts in this set. Good tenants prefer the new contract to s_0 and bad tenants may or may not prefer the new contract. In either case, the contract is profitable to offer since it lies above AC*(q) and $r + \delta$ and, thus, s_0 cannot be a Nash equilibrium.

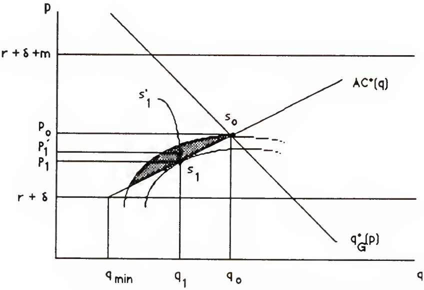


Figure 2.4. Contract s_0 not an equilibrium.

Contracts which lie below s_0 along AC*(q) imply rationing because they lie below the demand function. It is clear that tenants who expend $e(q_0)$ at s_0 prefer some contract with rationing. Landlords can attract these tenants by offering a profitable contract. In figure 2.4, good tenants prefer $s_1' = (p_1', q_1)$, for example, to s_0 and landlords prefer to offer s_1'. This is true whether or not bad tenants also prefer s_1'. Now, continuing to restrict our attention to single-contract sets of offers, competition would then force the market to zero profits at contract s_1 in figure 2.4. Only at s_1 is it not possible for another single-contract offer to break a potential equilibrium at s_1. Contract s_1 has the property that the utility of good tenants is maximized subject to the zero profit and single-contract offer constraints. Note that at s_1 there are fewer bad tenants and the benefits are passed on to the good tenants.

The following results consider only sets of contracts where "pooling" occurs. A pooling set of contracts exists when each contract in the set is preferred by at least one of each type of tenant. That is, both good and bad tenants choose each contract in the set.

Lemma 2.4. If we restrict the sets of contracts to contain pooling at each contract in the set, then a Nash equilibrium set of contracts, if it exists, must consist of one contract.

Proof. Lemma 2.4 follows directly from case (3) in the proof of Lemma 2.3, which relied on Lemma 2.1, the single-crossing property. Q.E.D.

Proposition 2.2. If a Nash pooling equilibrium exists, it must be characterized by quality rationing to both good tenants and bad tenants.

Proof. By Lemma 2.4, a pooling equilibrium can have only one contract and by Lemma 2.2, a pooling equilibrium must lie on $AC^*(q)$. By Proposition 2.1, s_0 in figure 2.4 cannot be a pooling equilibrium. No contract along $AC^*(q)$ above the demand curve can be an equilibrium, since s_0 is strictly preferred by good tenants to any such contract. Only contracts along $AC^*(q)$ below the demand function are potential pooling equilibrium contracts. These are all rationing contracts for good and bad tenants. Thus, if a Nash pooling equilibrium exists, it must entail quality rationing to both good and bad tenants. Q.E.D.

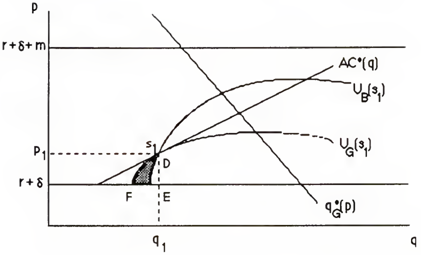


Figure 2.5. Contract s_1 not a Nash pooling equilibrium.

In figure 2.5, contract s_1 is obviously the only candidate for a pooling equilibrium. In general, however, s_1 is not a Nash equilibrium. There may exist a single-contract which is profitable to offer or there may a multiple-contract separating offer which is profitable to offer, given s_1 continues to be offered . Next, I will show that s_1 can never be a Nash equilibrium.

Proposition 2.3. Contract s_1 can never be a Nash pooling equilibrium.

Proof. Contract s_1 will not be a Nash pooling equilibrium if there exists a set of contracts which, when offered in addition to that contract, earn positive profits for each contract in the set. I will show that such a set exists. I will restrict attention to include only single contract sets, since that is all that is necessary to prove Proposition 2.3.

Let $s'=(p',q')$ be a contract offer such that $r + \delta < p' < p_1$, $q_{min} < q' < q_1$, $U_G(s') \geq U_G(s_1)$, and $U_B(s') < U_B(s_1)$. We know from the single-crossing property (Lemma 2.1) that at s_1 the iso-utility curve for the bad tenant intersects the iso-utility curve for the good tenant from below. Since $p_1 > r + \delta$, it is possible to choose s' to satisfy these conditions. If $U_G(s') \geq U_G(s_1)$ and $U_B(s') < U_B(s_1)$, then only good tenants prefer s' to s_1 and, since $p' > r + \delta$ it is profitable to offer. Thus, s' breaks s_1 as a Nash equilibrium. Q.E.D.

It is interesting to examine graphically the set of single-contract offers which can break s_1 as a Nash equilibrium. In figure 2.5, this set is represented by the shaded area within the triangular region DEF. Any contract offer in DEF will always be preferred by good tenants, those who have $T \geq e(q_1)$. Other tenants with $T < e(q_1)$ who prefer a contract in that set may become good or stay bad. Only those tenants who prefer that contract and will remain bad can render that contract unprofitable to offer. In figure 2.5, as long as the contract lies to the left of $U_B(s_1)$, this will not occur. This allows us to specify the set of single-contract offers which can break s_1 as a Nash equilibrium. Let $S' = \{s' \mid U_G(s') \geq U_G(s_1), U_B(s') < U_B(s_1), r + \delta < p' < p_1, q_{min} < q' < q_1\}$. In the proof of Proposition 2.3, S' was established to be nonempty. In figure 2.5, this set is the shaded area within DEF, including contracts along $U_G(s_1)$ to the left of s_1 and not including contracts along $U_B(s_1)$ to the left of s_1.

Proposition 2.3 would seem to eliminate the possibility of a pooling equilibrium in this model. However, the Nash concept of equilibrium may be inappropriate since, if contract s' exists, the shift of good tenants to s' would leave landlords at s_1 with only bad tenants and, consequently, with losses. Contract s_1 would disappear and s' would have to be used by bad tenants as well as good tenants. This implies losses for landlords at s'.

If we consider an alternative to Nash concept of equilibrium, however, s_1 may be a pooling equilibrium. In the alternative definition, due to Wilson (1977), s' will break s_1 as an equilibrium only if s' is profitable given s_1 will be withdrawn. If we consider only pooling contracts, then s'cannot break potential equilibrium at s_1. This does not imply that s_1 will be a pooling equilibrium, even with Wilson's definition. The reason is that there may exist a separating set of contracts which is profitable to offer whether or not s_1 is withdrawn.

Next I consider the possibility of a separating equilibrium and, if it exists, its properties. A separating set of contracts has the property that one type of tenant prefers one contract in the set and the other type prefers another contract in the set. A separating equilibrium set of contracts must again satisfy the necessary and sufficient conditions given by definition 2.1. Using arguments similar to those used in the proof of Lemma 2.3, it can be shown that a separating equilibrium must contain only two contracts. We can also say more about the price in each of these contracts.

Lemma 2.5. A separating equilibrium set of contracts must contain one contract, that attracts only bad tenants, with the price equal to $r + \delta + m$ another contract, that attracts only good tenants, with price equal to $r + \delta$.

Proof. Lemma 2.5 follows immediately from Lemma 2.2. Q.E.D.

Of course, good tenants pay $r + \delta$ and bad tenants pay $r + \delta + m$. Note that one important implication of Lemma 2.5 is that bad tenants will always be on their demand curve in a separating equilibrium. Any other contract offered to bad tenants with price equal to $r + \delta + m$ will allow for an alternative profitable contract, which attracts bad tenants, to be offered. This means that quality is not rationed to bad tenants in a separating equilibrium. Also, bad tenants must prefer to reveal themselves as bad tenants rather than mimic good tenants at $r + \delta$. That good tenants must not want to mimic bad tenants must also hold. These are the so-called self-selection constraints.

Proposition 2.4 below, along with Proposition 2.2, provide the primary result of this chapter that, in equilibrium, good tenants are always worse off in this asymmetric information setting than they would be in a world of perfect information.

Proposition 2.4. If a separating Nash equilibrium exists, it must entail quality rationing for good tenants.

Proof. From Lemma 2.4 we know that good tenants must pay a price equal to $r + \delta$. If it can be shown that good tenants can never obtain a contract which has this price and lies on their demand curve $q = q_G^*(p)$, then using the single-crossing property it follows that good tenants must obtain a quality level which lies to the left (or below) their demand curve.

Referring back to figure 2.3, all that must be shown is that when $q_G^* < q_B^*$ it is not possible to have a separating equilibrium with contracts (s_G^*, s_B^*), since if $q_G^* \geq q_B^*$, from Lemma 2.1, it is obviously not possible to satisfy the self-selection constraints. I will show this by contradiction. Assume that (s_G^*, s_B^*) is a separating equilibrium. This implies

(a) $I - (r + \delta + m)q_B^* - T + U(q_B^*) \geq I - (r + \delta)q_G^* - T + U(q_G^*)$.

From the assumption that $e(q) < mq$ for every q we have

(b) $I - e(q_B^*) - (r + \delta)q_B^* + U(q_B^*) > I - (r + \delta + m)q_B^* - T + U(q_B^*)$

for the bad tenant. For the good tenant, we have

(c) $I - (r + \delta)q_G^* - e(q_G^*) + U(q_G^*) > I - (r + \delta)q_B^* - e(q_B^*) + U(q_B^*)$,

since s_G^* lies on the demand curve. Now, the bad tenant must get at least $I - (r + \delta)q_G^* - e(q_G^*) + U(q_G^*)$ at $(r + \delta, q_G^*)$.

He gets exactly this utility if he is a good tenant at $(r + \delta, q_G^*)$. If $T < e(q_G^*)$ then his utility is even greater. Thus, we have

$I - (r + \delta)q_G^* - T + U(q_G^*) > I - (r + \delta)q_G^* - e(q_G^*) + U(q_G^*)$ and, from (b) and (c),
$I - (r + \delta)q_G^* - T + U(q_G^*) > I - (r + \delta + m)q_B^* - T + U(q_B^*)$.
This is a contradiction of (a). Thus (s_G^*, s_B^*), $s_G^* < s_B^*$ cannot be a separating equilibrium. Now, since good tenants can never get $(r + \delta, q_G^*)$ in a separating equilibrium, we know from the single-crossing property that they must obtain $(r + \delta, q)$, where $q < q_G^*$, in a separating equilibrium. This is exactly the result that there will be quality rationing for good tenants in a separating equilibrium, if it exists. Q.E.D.

In figure 2.6, for the purpose of illustrating some of the properties described above I have drawn a situation where a separating equilibrium exists. I do this without regard to the restrictions which can provide the configuration of curves shown. I shall reserve a discussion of existence for the next section.

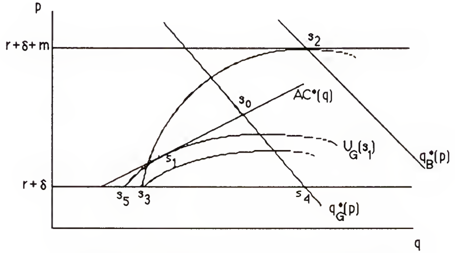


Figure 2.6. Example of a separating equilibrium.

With the breakeven constraint, landlords can offer any single-contract on $AC^*(q)$ or any combination of contracts with one each on the the horizontal lines $r + \delta$ and $r + \delta + m$. Clearly, the single-contract along $AC^*(q)$ most preferred by good tenants is s_1. We have already considered this case. If we offer the combination (s_2,s_4) then bad tenants would prefer to mimic good tenants and choose s_4 and losses would result (this was established in the proof of Proposition 2.4). If we offer (s_2,s_5) bad tenants prefer s_2 to s_5, but clearly there is contract along $r + \delta$ which is preferred by good tenants. A landlord can offer a slightly higher price and higher quality and find it profitable, but competition forces the price back to $r + \delta$. However, if we offer (s_2,s_3) bad tenants are just indifferent between s_2 and s_3 and there does not exist a profitable contract with q larger than in s_3. This is because bad tenants as well as good tenants would prefer it, resulting in losses. In effect a bad tenant is willing to reveal himself as a bad tenant and is willing to pay for the extra damage he causes by paying a higher price. Good tenants prefer to remain good but are willing to accept a lower quality in exchange for paying only average cost $r + \delta$. The result is that we still have quality rationing for good tenants while bad tenants can obtain enough quality to satisfy their demand at the higher price. It is somewhat counter-intuitive that a landlord is willing to rent a higher quality unit to a bad tenant, but remember that the bad tenant is willing to pay for the additional damage he causes.

2.5: Existence of a Nash Equilibrium

In this section, I shall consider whether or not a Nash equilibrium exists in the competitive rental housing market model developed in the preceding sections. I will show that a separating Nash equilibrium obtains under certain conditions.

It is necessary to define four contracts which will be used to examine whether or not a Nash equilibrium exists.

Contract s_1 maximizes the utility of good tenants subject to the constraint that the contract is on $AC^*(q)$. This gives the utilities

$$U_G(s_1) \equiv \max_{p\ q} [I - pq - e(q) + U(q)],$$

$$\text{subject to } p = (r + \delta) + (1 - \lambda)m$$

for good tenants and $U_B(s_1)$ for bad tenants.

Contract s_2 maximizes the utility of bad tenants subject to the constraint that $p = r + \delta + m$. This gives utility

$$U_B(s_2) \equiv \max_{q} [I - (r + \delta + m)q - T + U(q)].$$

Contract s_3 gives the bad tenant a utility of $U_B(s_2)$ at a price of $r + \delta$. Thus, $U_B(s_3) \equiv U_B(s_2)$ by definition.

Contract s_i is the contract which makes

$$U_G(s_1) = U(s_i) \text{ and } U_B(s_2) = U_B(s_3) = U(s_i).$$

Figures 2.7a - 2.7e represent five different possibilities for locating the above contracts.

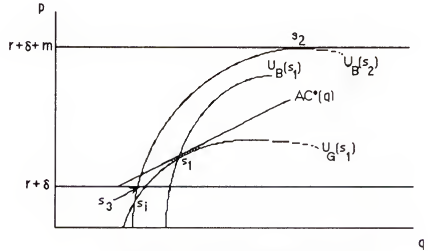
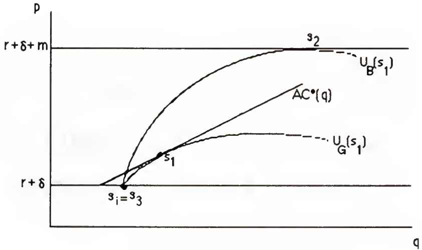
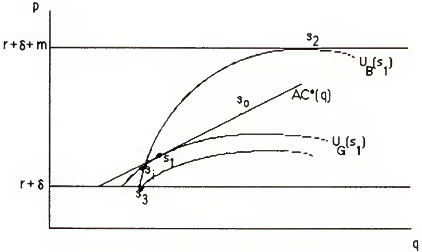


Figure 2.7. Existence of equilibrium.
a) No Nash equilibrium.

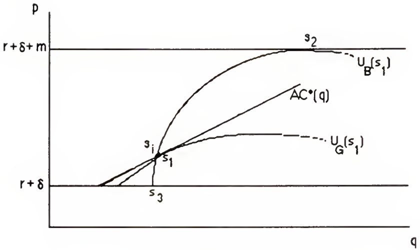


b) Separating equilibrium.

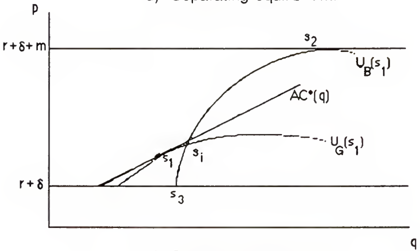


c) Separating equilibrium.

Figure 2.7--continued



d) Separating equilibrium.



e) Separating equilibrium.

Figure 2.7--continued

These five figures can be summarized as follows:

(2.13a) $U_B(s_2) = U_B(s_3) < U_B(s_1)$ and $p_i < r + \delta$

(2.13b) $U_B(s_2) = U_B(s_3) < U_B(s_1)$ and $p_i = r + \delta$

(2.13c) $U_B(s_2) = U_B(s_3) < U_B(s_1)$ and $p_i > r + \delta$

(2.13d) $U_B(s_2) = U_B(s_3) = U_B(s_1)$ and $p_i > r + \delta$

(2.13e) $U_B(s_2) = U_B(s_3) > U_B(s_1)$ and $p_i > r + \delta$.

These represent all possible configurations of the contracts and curves which we must consider. In this model, if $U_B(s_3) \geq U_B(s_1)$ it is not possible for $p_i \leq r + \delta$. Thus, we must consider only the above five possibilities.

Proposition 2.5. In Case a, a Nash equilibrium does not exist. In Cases b-e, a Nash separating equilibrium exists.

Proof. I will take each separately and show that in each case, except Case a, a Nash equilibrium exists

Case a (figure 2.7a, equation 2.13a). From Lemmas 2.4 and 2.5, only contract sets consisting of either one contract or two contracts are candidates for equilibrium. From Lemma 2.5, in order for a two contract set to be an equilibrium, bad tenants must purchase s_2 and good tenants must purchase a contract with $p = r + \delta$. The only contract set that satisfies both of these is the set (s_2,s_3). However, (s_2,s_3) cannot be a Nash equilibrium because there exists a single contract set which is profitable to offer. For example, there exists a contract with the same quality, but slightly higher price than the contract s_1, which is profitable to offer. This is because it can attract good and bad tenants and still be profitable to offer, since it lies above $AC^*(q)$. The only other candidate for a Nash equilibrium is s_1. However, in Proposition 2.3 it was shown that s_1 cannot be a Nash equilibrium. Thus, there does not exist a Nash equilibrium in Case a.

Case b (figure 2.7b, equation 2.13b). The set (s_2,s_3) is a Nash equilibrium. To break this set as a Nash separating equilibrium there must exist at least one contract which is profitable to offer given (s_2,s_3) continues to be offered. From Lemma 2.3, I need only consider single contract offers which can break (s_2,s_3) as a Nash equilibrium. Any contract s'' which is profitable to offer must (a) attract only good tenants and lie above $r + \delta$ or (b) attract both good tenants and bad tenants and lie above $AC^*(q)$. The restrictions in (a) cannot be satisfied since if $p > r + \delta$ and $U_G(s_3) \leq U_G(s'')$, then $U_B(s_2) < U_B(s'')$ by the single-crossing property. Also, since $U_G(s_3) > U_G(s_1)$ the restrictions implied by (b) cannot be satisfied. Thus, there does not exist a contract which is profitable to offer given (s_2,s_3) continues to be offered. That is, (s_2,s_3) is a Nash equilibrium.

Case c (figure 2.7c, equation 2.13c). The set (s_2,s_3) is a Nash equilibrium. The proof is identical to that for Case b.

Case d (figure 2.7d, equation 2.13d). The set (s_2,s_3) is a Nash equilibrium. The proof is identical to that for Case b.

Case e (figure 2.7e, equation 2.13e). The set (s_2,s_3) is a Nash equilibrium. The proof is identical to that for Case b. Q.E.D.

In cases b, c, d, and e, a Nash separating equilibrium exists. Of course, it was already shown that there does not exist a Nash equilibrium in Case a. It does seem possible to find conditions on the distribution of T and the constant difference in costs, m, which restrict the possibility of Case a.

Although a Nash equilibrium does not exist in Case a, s_1 may be a pooling equilibrium if we employ the Wilson definition. Clearly, no single contract offer can break s_1 as an equilibrium according to this alternative definition. This was discussed in Section 2.4. However, there may be multiple contract offers which can break s_1 as a potential equilibrium, even with the alternative definition.

2.6: Conclusions

In this chapter I have analyzed landlord and tenant behavior and the characteristics of equilibrium in a rental housing market which takes into account that tenants have more information about their likelihood of maintaining a dwelling than landlords do. The landlord response to this informational asymmetry will always make good tenants worse off relative to the perfect information case. If a Nash equilibrium exists, good tenants are always rationed in terms of the level of quality they can obtain.

CHAPTER 3
DOUBLE MORAL HAZARD IN A TIME-DEPENDENT SUPERGAME MODEL OF RENTAL HOUSING

3.1: Introduction

In this chapter, I will employ the ideas of double moral hazard and trigger strategies to analyze subgame perfect equilibria in an infinite-horizon time-dependent supergame model of rental housing. Both the landlord and tenant take actions which affect the quality of the unit of housing. Time-dependence is introduced by allowing current quality to depend on quality in the previous period as well as the actions of the tenant and landlord. In any period, the history of quality is observable to everyone and verifiable by the courts. Since there is no stochastic component affecting quality, the landlord and tenant can infer what the other has chosen in the past. However, the individual choices of the landlord and tenant cannot be verified by the courts. This is the source of the double moral hazard problem. The tenant and landlord will have an incentive to choose inefficient actions.

I will show that it is possible to design a contract such that efficient choices by both the tenant and landlord can be supported in a subgame perfect Nash equilibrium. To do this, I will employ strategies which contain credible and effective threats, which are used if the landlord or tenant deviates from the efficient choices. The idea is that any deviation "triggers" a response which causes both to be strictly worse off than if the deviation did not occur. The contract divides the surplus between the landlord and tenant, both on and off the equilibrium path, such that these threats are credible.

The organization of this chapter is as follows. In Section 3.2, I survey the most related literature concerning moral hazard, rental housing markets, repeated games, and time-dependent supergames. Many ideas and terms relevant to this chapter are introduced in this section. In Section 3.3, the notation and assumptions of the model are presented. In Section 3.4, the Pareto-efficient paths of effort and maintenance choices are developed with respect to any initial quality. I prove that starting from any initial quality, there will be convergence to a unique steady-state quality. In Section 3.5, I intoduce a simple rental contract and characterize a subgame perfect Nash equilibrium of the supergame. I call this equilibrium the "grim equilibrium." In Section 3.6, I make the assumption that we begin with the unique Pareto-efficient steady-state quality and construct a proof that this quality can be sustained as a subgame perfect Nash equilibrium with efficient choices of effort and maintenance. To support this equilibrium I will use the "grim equilibrium" strategies introduced in Section 3.5 as the threat component of trigger strategies. In Section 3.7, I discuss the results with respect to other equilibria. I also examine the extension of this model to allow for renegotiation or a finite horizon. In the final section, I provide some concluding remarks.

3.2: Survey of the Literature

It has long been recognized that rental housing markets are characterized by a moral hazard on the part of the tenant. One version of the standard principal-agent setup would have the landlord as the principal and the tenant as the agent. The landlord's payoff depends on an action taken by the tenant. They start with symmetric information and agree to a contract, but then the tenant takes an action unobserved by the landlord. In rental housing relationships, the action is the effort put forth by the tenant to maintain and care for the apartment he occupies. Assuming the tenant will leave the unit at some point, the moral hazard is that the tenant does not have an incentive to behave efficiently because the benefit accrues only to the landlord through higher future rents. Another version of the principal-agent setup would have the landlord and tenant begin with symmetric information, but then "nature" takes a move observed by the tenant but not the landlord. Many feel that this is really just a form of adverse selection. In this chapter, I will use the former setup in modelling moral hazard. See Harris and Raviv (1979) for the original discussion of the different setups. See Rasmusen (1989) for a discussion of the circumstances when each approach is appropriate.

Henderson and Ionnides (1983) emphasize the problem of tenant moral hazard in their model of tenure choice (whether the household owns or rents the dwelling it occupies). They call it a rental externality. Relative to owner occupancy, excessive deterioration of the unit will occur if occupied by a renter. Kanemoto (1990) recognizes the importance of the externality in determining the structure of contracts in housing markets. He identifies the source of this externality as the lack of third-party verification and concludes that, since the tenant's effort cannot be verified in court, a rental contract cannot yield the first-best (efficient) allocation of housing investment. The introduction of tenant moral hazard, thus reduces the set of efficient contract forms to include only the purchase contract. He also introduces the idea that when analyzing rental contracts the security of tenure for the tenant is important. In the introductory chapter of Microeconomic Models of Housing (1985), Konrad Stahl discusses not only the typical problems associated with modelling rental housing markets (for instance, the problems of durability and differentiability of housing), but also some very important informational problems. He emphasizes these problems in the context of adverse selection — tenants are of different types with characteristics like propensity to maintain the unit, sociableness, and payment morale unknown to the landlord a priori. He notes that, since many of these traits reveal themselves over time, this may lead to discounts to good tenants who renew their rental contract. He also adds that these problems may lead to some price rigidity and perhaps some sort of demand rationing in rental housing markets.

As Kanemoto (1990) notes, the source of the tenant moral hazard is not necessarily that the landlord cannot observe the behavior of the tenant. The landlord may make frequent inspections or may take a careful inventory of the condition of the unit before and after each period of occupancy. The problem is that the tenant's effort is not perfectly verifiable (or too costly to verify) by the courts. It may be that nature, or even the landlord, can also have an effect on the quality of the unit, and the courts cannot separate these effects from the behavior of the tenant.

There appears to be many mechanisms which can be used to solve this problem. The tenant may be required to pay a deposit, in addition to the rent, at the beginning of the rental period. The end-of-period quality is compared with the initial quality to determine the extent of the damage done to the unit and the "appropriate" portion of the deposit is withheld by the landlord. This would be similar to workers posting bonds in labor markets characterized by moral hazard. Although deposit requirements are widely used, there is a serious problem with their ability to solve the moral hazard problem. The landlord will generally have an incentive to withhold the full amount of the deposit, given the imperfect nature of verification. Also, imposing a large deposit requirement may not be possible because of wealth constraints. The role of damage deposits in rental housing markets is examined in Chapter 4 of this dissertation.

It is possible that if the tenant were to occupy the unit forever, the moral hazard problem can be solved. The incentive to put forth efficient effort comes from the future benefits of higher quality. Although this solution appears to be simple, there are several complicating factors which must effectively be dealt with. For instance, in this chapter, the quality of the unit will also depend on maintenance by the landlord. The inability to verify this maintenance allows the possibility of landlord moral hazard, as well as tenant moral hazard. Constructing a model where the tenant remains in the unit forever will make this problem even more serious, since the landlord need not worry about reconditioning or marketing a vacant unit. This is illustrated by Kanemoto (1990) where, in his model, a rental contract with perfect security of tenure for the tenant will result in inefficient investment by the owner. It is only when there is no security of tenure that the owner's choices are efficient.

With landlord maintenance and tenant effort both affecting quality, the problem becomes one of double moral hazard. Double moral hazard in static models has been addressed by many authors. Two of the earliest papers dealing with this problem are by Holmstrom (1982) and Kambhu (1982). Holmstrom's model deals with moral hazard in teams, defined loosely as a group of individuals organized so their productive inputs are related. The idea is that when the joint output of a "team" is the only observable indicator of inputs, members of the team who cheat (make inefficient choices) cannot be identified. The result is, of course, Pareto inefficiency. In Kambhu's model a seller provides a product with unobservable innate quality, which along with the buyer's care of that product affects the observable measure of product performance. Although performance can be observed, it is impossible to attribute the performance of that product to either the seller's choice of quality or the buyer's choice of care.

In both of these models, if a "balanced budget" is required, Pareto efficient choices will not obtain. A balanced budget constraint imposes the restriction that the transfer mechanism is designed such that the sum of the payoffs to all the players exactly equals the value of the collective product. In the Holmstrom model, if the team output must equal the sum of the rewards to its members, each member has an incentive to "free ride" by choosing an inefficient action. In the Kambhu model, if the price received by the seller must equal the price paid by the buyer, it is not possible to obtain efficient choices of both quality and care simultaneously.

If a balanced budget is not required it may be possible to obtain efficient behavior. By introducing a principal (who provides no productive inputs) to assume the residual (output minus the sum of the rewards) when output is below the efficient amount, Holmstrom shows the that the free-rider problem can be eliminated. The share to each worker can be zero if output falls short of the efficient amount. If it does not, the sum of the shares can be set just equal to output. This essentially allows for punishment by the principal if any cheating is detected. In Kambhu's model, if the performance of the product is not efficient, the buyer does not receive compensation and the seller pays a penalty to a third-party participant. The price actually received by the seller is then not what the buyer pays, allowing for punishment for inefficient choices. Thus, efficient quality and care can obtain. However, the usefulness and realism of such "unbalanced" mechanisms has been questioned. There will always be incentives for the third-party involved to form covert agreements with the agents. As Eswaran and Kotwal (1984) point out, introducing a third party to break the budget-balancing constraint simply causes the problem of moral hazard to take a different form.

There are other models with double moral hazard that retain the balanced budget constraint. Cooper and Ross (1985) focus on the effect of double moral hazard in shaping warranty contracts. The buyer is unable to observe quality and the seller cannot observe effort by the buyer, a priori. Both affect the probability that the product breaks down. The standard inefficiency result obtains. However, second-best equilibrium distortions depend on whether effort and quality are substitutes or complements in reducing the probability of product breakdown. Mann and Wissink (1988) analyze double-moral hazard in the context of money-back guarantees. Although the model relies on some uncertainty (but not too much) to obtain first-best choices, it does economize on the information required by the contract between buyer and seller. Essentially, no verification of product performance is necessary. Only a record of the purchase terms and verification of the return must be required. This model is particularly interesting because, although the payment mechanism is balanced, a return involves the loss of surplus. This means that the budget balancing constraint is relaxed for the contract as a whole. In this chapter, we will find that the loss of surplus off the equilibrium path allows the first-best choices of effort and maintenance to support the efficient steady-state quality level, even though the payment mechanism is balanced. The key difference is that, in this chapter, the loss of surplus occurs in the future. Mann and Wissink (1988) derive their results in a static setting. Finally, Eswaran and Kotwal (1985) view sharecropping as a partnership arrangement, and share contracts emerge to mitigate moral hazard problems on the part of both landlord and tenant.

Models with double moral hazard have also been used to analyze vertical relationships and contracts. Both the upstream firm and downstream firm can take actions which affect the final demand for the product. Romano

(1991) shows that the resulting optimal contract may involve maximum or minimum resale price maintenance. Mathewson and Winter (1985) analyze royalty contracts in franchising arrangements with double moral hazard.

It is obvious that there are many markets characterized by double moral hazard. However, even though it has been noted that there can exist a tenant or landlord moral hazard, there has never been a systematic application of double moral hazard to analyze relationships and contracts in rental housing markets.

It is clear from the above review that in any market characterized by moral hazard, the contract must depend only on verifiable variables. Not only must the variables be observable to both parties, but they also must be perfectly verifiable by the courts. See Hart and Holmstrom (1987) for a survey of the theory of contracts. In Holmstrom's model, team output was perfectly and costlessly observable by the third-party. In Kambhu's model, the measure of quality was verifiable by everyone, including the third party. In Cooper and Ross (1988), the actual performance of the product was observable. In Mann and Wissink (1988), only the terms of the purchase and actual return need be verifiable. In each case, the nature of what is verifiable determines the nature of the optimal contract. In this model, the actual behavior of tenant and landlord are not contractible or else there would be no problem of moral hazard. However, the choice of effort and the choice of maintenance affect the measure of quality which is verifiable at the beginning of the next period. Note that since nature does not affect quality, the tenant and landlord can infer the actual past behavior of the other. Each can determine what the other has chosen because they can observe quality and know, of course, their own past choices. I assume the contract, which we know cannot specify behavior, consists of the rent paid by the tenant to the landlord, contigent upon the history of quality, which is verifiable. To analyze this, we require a dynamic model. Therefore, I think it is useful to review some of the relevant literature on dynamic games.

When an explicitly dynamic model is introduced there are many questions which must be answered. Is the game to take place over a finite or infinite number of periods? Should agents discount future periods? Should payoffs in each period depend on only current behavior or on past behavior as well? Can actions taken in any period be restricted to be independent of information that becomes available up to that period? How is equilibrium defined and are there any refinements that seem natural? I will discuss each of these questions with respect to both the existing literature and this model.

The term "supergame" refers to a model where a sequence of games is played by a fixed number of players. The number of sequences played can be finite or infinite. Players may or may not discount future payoffs. Note that although the term "supergames" was introduced to describe only infinitely repeated games with no discounting, this distinction has become blurred over time. As we shall see below, the choice of finite vs. infinite horizon and whether or not to allow discounting are very important in determining the number and types of equilibria which result in any model. If the same game is played in each period, the supergame is called a "repeated game." In these models the payoff-relevant circumstances of the initial time period repeat themselves identically in each succeeding time period. That is, each of the players face the same "stage game" or "constituent game" in each period. These games can be contrasted with "time-dependant supergames," where the payoff structure in any period depends on actions taken in that period and earlier periods. The solution concept commonly used in these models is the Nash equilibrium concept. The most interesting result of the introduction of supergame models is that equilibria which are not possible in single period (single-shot, static) games can become equilibria if modelled as a supergame. Sometimes these newly obtainable equilibria can include Pareto-efficient choices of strategy. The reason for this result is that, in supergame models, it becomes possible to condition choices on observed actions from previous periods. Notice that even if explicit (structural) time-dependence is not introduced (eg. repeated games) in the model, there is some strategic time dependence, since actions in any one period can depend on the history of actions up to that time period.

The initial formal work done in this area was concerned with infinitely repeated games with no discounting. For example, consider the prisoners dilemma, which in a static game has a dominant strategy equilibrium with both players finking. Also, using backwards induction it is easy to show that even if the game were repeated a known finite number of times, the only Nash equilibrium calls for each player to fink in each period. It can also be shown that finking every period is also the only subgame perfect Nash equilibrium in the finitely repeated prisoners dilemma. Subgame perfection will be introduced below. However, if the game were repeated over an infinite horizon with no discounting, it can be shown that Pareto-efficient cooperating (each choosing to hold out) can be sustained even as a noncooperative equilibrium. Cooperation is sustained by means of a threat to fink forever if one of the players ever chooses not to cooperate. This is an example of an equilibrium in "trigger strategies." It is called this because if one player chooses to deviate from cooperation it "triggers" the use of the static equilibrium choices of finking forever. Employing the static choices forever in response to another's deviation is often called a "grim" trigger strategy. This result can even obtain with a discount factor less than but sufficiently close to one. In fact, these results can be generalized as in the so-called Folk Theorems. (See Fudenberg and Maskin, 1986.) Generally, these folk theorems state that outcomes where each player receives payoffs which exceed the minimum payoff to which that player can be held can be supported as the outcome of a noncooperative Nash equilibrium.

The problem in these models is that the set of equilibria is enlarged considerably by the introduction of an infinite horizon and history-dependent strategies. In some cases it may not be unreasonable to assume the players coordinate on a "focal equilibrium". Most often this focal equilibrium is assumed to be Pareto optimal. Alternatively, one can consider some "refinements" to the Nash definition of equilibrium, which were developed to eliminate some equilibria which may seem intuitively unreasonable. The most popular refinement is the additional requirement of subgame perfection due to Selten (1965,1975).

In a repeated game, a subgame is defined as the play of the game from any given time period t, onward, together with the history of the game at time t. By specifying a history, one specifies a subgame. A subgame is usually defined in terms of information nodes. Specifically, a subgame is a game consisting of a node which is a singleton in every players information partition, that nodes' successors, and the payoffs at the associated end nodes. However, it is not difficult to show that the two definitions are equivalent in the repeated game framework. I will not show this, but I refer the reader to Friedman (1990, pp. 118-119). Imposing the requirement of subgame perfection means that an equilibrium point must be an equilibrium in every possible subgame, even if the subgame defined by a history is never actually encountered in equilibrium. Subgame perfection is especially important in these models because trigger strategies involve the use of threats to play punishment strategies for certain histories. Although one player may never be required to follow through with a threat, unless the threat is credible it may be ineffective. Subgame perfection renders threatened strategies credible by requiring that they be equilibrium strategies at the points where they are to be played. The requirement of subgame perfection does not eliminate any equilibria of the repeated prisoners dilemma. In fact, the trigger strategy equilibria introduced above can be strengthened to include subgame perfection. Aumman and Shapley (1976), Rubenstein (1979), and Fudenberg and Maskin (1986,1990) have proven that equilibrium strategies in the folk theorem can be selected to be subgame perfect for a range of discount factor sufficiently close to one. The folk theorem usually describes the behavior of the set of perfect equilibria as the discount factor approaches one. However, Abreu (1986, 1988) considers the set of perfect equilibria for a fixed discount factor less than one. In the process, he constructs strategies such that any deviation by a player is punished by play switching to the perfect equilibrium in which that player's payoff is lowest. In fully general time-dependent supergames, the idea of subgame perfection must be extended to include subgames introduced by different initial values of the state variable, the variable which allows the players' current choices to affect payoffs in future periods. (See Fudenberg and Tirole, 1986.) As we shall see later, in models where history matters directly, the requirement of subgame perfection becomes even more important. For this reason, and since the notion of subgame perfection is intuitively appealing when threats are used, I will retain it as a requirement in this chapter.

As noted above, only the static equilibrium obtains in a finitely repeated prisoners dilemma game. However, the equilibria obtained in the infinite horizon model can also be obtained in models where there is a constant and sufficiently small probability that each period is the last. In this case, the game does end with probability one. Also, although it was thought for some time that the only subgame perfect equilibrium in a finitely repeated game is a static equilibrium, this is not true in general. In the prisoners dilemma game it is true only because finking is the unique static equilibrium. It is also true if all static equilibria have the same payoff. However, when both of these are not true, Friedman (1985), with Rawthorne's (1988) correction, has proven the existence of subgame perfect trigger strategy equilibria. Also, for such games, Benoit and Krishna (1985) prove a version of the folk theorem with subgame perfection in finite-horizon repeated games with no discounting. Radner (1980) takes a different approach to showing the existence of trigger strategy equilibria in finite-horizon repeated games. He uses a bounded rationality approach by employing what he calls ε-equilibrium. An alternative approach which allows there to be a small probability that the players get extra satisfaction for cooperating, so long as their opponent has cooperated with them in the past is taken by Kreps, Milgrom, Roberts, and Wilson (1982). I will not employ either of these last two approaches in this chapter.

It is useful at this point to note that most of the results obtained thus far not only depend on the discount rate and time horizon, but also rely on the perfect monitoring of the actions taken in previous periods. For instance, we know that with perfect monitoring of the actions used by other players, an infinite horizon, and with no discounting, the folk theorem can obtain. (See Aumman, 1984.) With discounting, however, all equilibria may be inefficient unless it is a two-player game. In this case, Fudenberg and Maskin (1983) show that any efficient, individually rational payoff structure of the static game can be approximated by payoffs formed by a supergame equilibrium for discount factors sufficiently close to one. In fact, Fudenberg and Maskin (1990) show that in two-player games, when mixed strategies are allowed, the folk theorem can obtain with discounting, discount factors sufficiently close to one, and perfect monitoring of past actions. As we shall see below, similar results concerning sustainable equilibria can be obtained with imperfect monitoring, at least with respect to particular applications.

In the industrial organization literature, trigger strategy equilibria and the folk theorem have most often been applied in oligopoly models to help explain why collusion may be sustainable and when it is likely to break down. For instance, Radner introduces his ε-equilibrium approach in an oligopoly model where the result is that collusion becomes viable even with a finite horizon. Porter (1984) describes how collusive output levels can be maintained in a model where each firm can observe only its own production and the market price. Green and Porter (1984) develop a model of oligopoly collusion where cheating is not perfectly observable. Rotenberg and Solaner (1986) analyze collusion with fluctuating demand. See Tirole (1989), Chapter 6, and The Handbook of Industrial Organization, Chapters 6 and 7, for more on these models.

Radner (1981) again uses his ε-equilibrium to show that Pareto-optimal cooperative agreements can obtain as non-cooperative equilibrium in a principle-agent relationship (single moral hazard and imperfect monitoring) if the finite horizon is long enough. In Radner (1986) double moral hazard in a repeated game is analyzed. In his partnership model, each players' utility depends on his action and the actions of the other players through a commonly observed variable with some stochastic component. He shows that with no discounting, and with each player employing a statistical test (introduced in Radner (1981) above) for cheating, efficient behavior can be sustained as a Nash equilibrium in an infinitely repeated game. However, in the same journal Radner et al (1986) provide an example where, with some discounting, such a result cannot obtain. Fudenberg, Levine, and Maskin (1988) provide conditions for the folk theorem to obtain in a model with double moral hazard and imperfect monitoring of the outcomes of the players' actions.

In most of the repeated games literature discussed above, notice that history matters only strategically. Each players strategy depends on the past only because the other players' strategies do. This is a sort of "bootstrapping" phenomenon. With a finite horizon, backwards induction does not allow such bootstrapping to support many supergame equilibria. Many perceive this "bootstrapping" to be a problem with the repeated game approach. Instead one should incorporate some reason for history to matter directly in the model. This motivated economists to study environments where the past has a direct influence on current opportunities. In doing this, economists often focus attention on "Markov" or "state-space" strategies in which the past influences current play only through its effect on a state variable that summarizes the direct effect of the past on the current environment. In this chapter, I will not restrict attention to Markovian strategies. However, the past does matter not only because players believe that it matters, but also because of its effect on the current state. Because of this, it is necessary to discuss some ideas concerning time-dependent supergame models.

Before moving on, we must clarify how the definition of the strategy space and its associated equilibria becomes important in time-dependent models. With what are called "open-loop" or "precommitment" strategies and equilibria, the players simultaneously commit themselves to entire time-paths of choices. This precommitment essentially transforms the game into a static one. This does not seem natural in most applications. With "closed-loop" strategies and equilibria, actions in any period can incorporate information which becomes available as the game progresses. Note that open-loop equilibria are also closed-loop equilibria so that allowing the strategies to be time-dependent, although intuitively appealing, has the effect of enlarging the set of equilibria.

Supergames can be modelled with time-dependent structures. The single-period payoff for a player at time t depends on current actions of all players and their actions in one or more previous periods. Friedman (1974, 1981, 1990a, 1990b) examines both trigger strategy equilibria and the folk theorem in the context of time-dependent supergames. He shows, for instance, that it is possible to develop sufficient conditions for the existence of trigger strategy equilibria. He also shows that trigger strategies may call for players to use open-loop equilibrium strategies and these are not subgame perfect. In a game defined by open-loop strategies, this does not mean that these equilibria contain noncredible threats. However, expanding the strategy space to include history-dependence would eliminate many open-loop equilibria from the set of credible threats. Even closed-loop equilibrium strategies may fail to be subgame perfect and, thus, may not be credible when used as threats. This is because a closed-loop strategy can depend on the history but need not be subgame perfect. The point is, subgame perfection must be checked for carefully to rule out empty threats.

Fudenberg and Tirole (1986) survey some the recent work on dynamic oligopoly models. Of particular interest for this chapter is their presentation of models of competition in "tangible" variables. For instance, payoffs in any one period may depend on some tangible variable like capacity. This capacity depends on investment in capital stocks in previous periods. However, with the "state-space" assumption introduced above, the way this capacity was acquired does not matter. The current payoff depends only on the level of the capital stock this period. In this chapter, quality in any period depends on choices by the landlord and tenant in all previous periods. Past choices matter because they affect current quality, but they also matter because the landlord and tenant condition their strategies on them. Fudenberg and Tirole's discussion is also useful because, based on Starr and Ho (1967), they draw a connection between these models and elements of dynamic programming. In this chapter, the fundamentals of deterministic dynamic programming are used to help characterize efficient choices and a subgame perfect "grim" equilibrium.

3.3: Assumptions and Notation

In any period $t=0,1,2,\ldots,\infty$, the quality, q_t, of a rental unit depends on the quality, tenant effort, and landlord maintenance last period (resp. q_{t-1}, e_{t-1}, m_{t-1}):

$$q_t = q_{t-1} + d(e_{t-1}, m_{t-1}).$$

I will often refer to this equation as the transition equation. Effort and maintenance affect next periods quality through the damage function $d(\cdot,\cdot)$. Note that d does not vary with respect to time. Also note that all the results derived in the chapter would go through if we wrote $d(e_{t-1}, m_{t-1}, q_{t-1})$ as long as we are in the steady-state where $q = q_{t-1} = q_t$. Assume d is concave with the following properties:

$$d \in D,\ D \equiv \{ d : d_{min} \leq d \leq d_{max} \} \subset \Re,$$

$$\text{where } d_{min} \equiv d(0,0) \text{ and } d_{max} \equiv d(\bar{e},\bar{m}),$$
$$\frac{\partial d}{\partial e_t} > 0,\ \frac{\partial d}{\partial m_t} > 0,\ \frac{\partial^2 d}{\partial e_t^2} \leq 0,\ \frac{\partial^2 d}{\partial m_t^2} \leq 0,\ \frac{\partial^2 d}{\partial e_t \partial m_t} \geq 0,\ \text{and}$$
$$\frac{\partial^2 d}{\partial e_t^2}\frac{\partial^2 d}{\partial m_t^2} - \left(\frac{\partial^2 d}{\partial e_t \partial m_t}\right)^2 \geq 0.$$

The $\bar{e}$ and $\bar{m}$ above will be defined below as the upper bounds in the set from which effort and maintenance can be chosen. By assuming the cross-partial derivative is nonnegative, we are restricting attention to the case where effort and maintenance are independent or complements in producing quality. Kambhu (1982) assumes that the choice variables are substitutes, while

Cooper and Ross (1988) look at each case, when they are substitutes, complements, and independent.

The history of quality from time zero to time t is denoted by h_t, where

$$h_t = \{q_0, q_1, \dots, q_t\}.$$

I will assume that this history is perfectly and costlessly verifiable by the courts at the beginning of period t. This will allow the contract between the landlord and tenant to depend on observed past quality.

From occupying a unit of quality q_t in period t the tenant receives utility $U(q_t)$, where $U'(q_t) > 0$, $U''(q_t) \leq 0$, $U'(0) = \infty$, and $U'(\infty) = 0$. The tenant also gets disutility $c(e_t)$ from putting forth effort, e_t, to increase quality in period t+1. Assume

$$e_t \in E \text{ where } E \equiv \{e : 0 \leq e \leq \bar{e} < \infty\} \subset \Re,$$

$$c'(e_t) > 0,\ c''(e_t) > 0.$$

The tenant also pays rent, $R(h_t)$, to the landlord for a unit with history h_t in period t. This rental function will be discussed in more detail when the second-best, informationally-constained problem is considered.

The landlords utility, net of payments, in period t is $-r(m_t)$, where m_t is the maintenance he performs in period t. Assume

$$m_t \in M \text{ where } M \equiv \{ m : 0 \leq m \leq \bar{m} < \infty\} \subset \Re,$$

$$r'(m_t) > 0,\ r''(m_t) > 0.$$

Of course, the landlord also receives rent $R(h_t)$ from the tenant. Again, the properties of R will be discussed in more detail later in this chapter.

Using the above notation, the utility for the tenant in period t is

(3.1) $U(q_t) - c(e_t) - R(h_t)$

and the utility for the landlord is

(3.2) $R(h_t) - r(m_t)$.

3.4: Pareto Efficiency

Given an initial quality, q_0, the Pareto efficient effort and maintenance in periods $t=0,1,2,\ldots,\infty$ are determined by maximizing the discounted sum of tenant and landlord utility subject to the transition equation constraint. Note that this is a problem where the utility is transferable so that the unique Pareto-efficient choices and quality can be determined by maximizing the sum of the utilities. This maximization also determines the efficient path of quality over time starting with any initial quality. I will examine only "balanced" payment mechanisms where $R(h_t)$ is the amount paid by the tenant and received by the landlord.

Let

$$W_t \equiv U(q_t) - c(e_t) - r(m_t).$$

Note that with our assumptions, W_t is strictly concave in $(e_0,e_1,\ldots,e_t,m_0,m_1,\ldots,m_t)$. The objective function in the infinite horizon problem can be written as

$$\text{(3.3)} \qquad \max_{e_t\, m_t} \left\{ \sum_{t=0}^{\infty} \delta^t W_t \right\}$$

subject to $q_{t+1} = q_t + d(e_t, m_t)$, $t = 0,1,2, \ldots, \infty$ and q_0 given.

The structure of this model allows the use of standard dynamic programming techniques. In particular, since effort and maintenance in period t affect W_s and q_{s+1} for $s \geq t$ but not earlier, the problem has a recursive structure. This allows e_t, m_t, q_{t+1} for $t=0,1,\ldots,\infty$ to be solved sequentially not simultaneously.

Bellman's equation becomes

$$\text{(3.4)} \qquad V(q_t) = \max_{e_t\, m_t} \left\{ W_t + \delta\, V(q_{t+1}) \right\}$$

where $q_{t+1} = q_t + d(e_t, m_t)$ and with q_0 given. The unique limiting value function V that solves (3.4) turns out to be the optimal value function for (3.3). (See Theorem A.1 in Appendix.) It can be shown that, under our assumptions V is strictly concave. (See Theorem A.2 in Appendix.) There exists unique and time invariant optimal policies $e^*(q_t)$ and $m^*(q_t)$ where e^* and m^* are chosen to maximize the right-hand side of (3.4). (See Theorem A.3 in Appendix.) This is a consequence of the infinite horizon and time-invariant utility and damage functions. The first-order conditions for this maximization are

$$(3.4a) \qquad - c'(e_t) + \delta V'(q_{t+1})\frac{\partial d(e_t,m_t)}{\partial e_t} = 0$$

$$(3.4b) \qquad - r'(e_t) + \delta V'(q_{t+1})\frac{\partial d(e_t,m_t)}{\partial m_t} = 0.$$

Furthermore, the limiting value function V is differentiable (see Theorem A.4 in Appendix) giving a version of the equation of Benveniste and Scheinkman

$$(3.5) \qquad V'(q_t) = U'(q_t) + \delta V'(q_{t+1}).$$

This equation will be used later to help derive first-order conditions which describe efficient choices at the efficient steady-state quality, which will be derived below.

Next, I will show that given our optimal policy functions there exists a unique value, q^*, of the state variable such that if $q_t = q^*$, then $q_{t+i} = q^*$, $i=1,2,\ldots,\infty$, is optimal. Also, starting from any $q_0 > 0$ we have convergence to this unique value. I shall call q^* the steady-state quality. Substituting the optimal policy functions into the transition equation we have

$$(3.6) \qquad q_{t+1} = q_t + d(e^*(q_t), m^*(q_t)),$$

a first-order (perhaps) nonlinear difference equation. Differentiating (3.6) with respect to q_t gives

(3.7) $$\frac{dq_{t+1}}{dq_t} = 1 + \frac{\partial d(e^*,m^*)}{\partial e}e^{*\prime}(q_t) + \frac{\partial d(e^*,m^*)}{\partial m}m^{*\prime}(q_t).$$

Lemma 3.1. $0 \leq \frac{dq_{t+1}}{dq_t} < 1$

Proof. I will show $0 \leq \frac{dq_{t+1}}{dq_t} < 1$ by using a direct proof.

Differentiating (3.4a) and (3.4b) with respect to q_t gives

(C) $$- c^{\prime\prime}e^{*\prime} + \delta V^{\prime}\frac{\partial^2 d}{\partial e_t^2}e^{*\prime} + \delta V^{\prime}\frac{\partial^2 d}{\partial e_t \partial m_t}m^{*\prime} + \delta V^{\prime\prime}\frac{\partial d}{\partial e_t}[\, 1 + \frac{\partial d}{\partial e_t}e^{*\prime} + \frac{\partial d}{\partial m_t}m^{*\prime}\,] = 0$$

and

(D) $$- r^{\prime\prime}e^{*\prime} + \delta V^{\prime}\frac{\partial^2 d}{\partial m_t^2}m^{*\prime} + \delta V^{\prime}\frac{\partial^2 d}{\partial m_t \partial e_t}e^{*\prime} + \delta V^{\prime\prime}\frac{\partial d}{\partial m_t}[\, 1 + \frac{\partial d}{\partial e_t}e^{*\prime} + \frac{\partial d}{\partial m_t}m^{*\prime}\,] = 0.$$

From (3.7), we know that the term in the brackets of each equation is simply the slope of $q_{t+1} = q_t + d(e^*(q_t)\,,\, m^*(q_t))$. Substituting $\frac{dq_{t+1}}{dq_t}$ into (C) and (D) and rearranging gives

$$e^{*\prime} = \frac{-\delta V^{\prime}\frac{\partial^2 d}{\partial e_t \partial m_t}}{\delta V^{\prime}\frac{\partial^2 d}{\partial e_t^2} - c^{\prime\prime}}\, m^{*\prime} - \frac{\delta V^{\prime\prime}\frac{\partial d}{\partial e_t}\frac{dq_{t+1}}{dq_t}}{\delta V^{\prime}\frac{\partial^2 d}{\partial e_t^2} - c^{\prime\prime}} \quad \text{and}$$

$$m^{*\prime} = \frac{-\delta V^{\prime}\frac{\partial^2 d}{\partial e_t \partial m_t}}{\delta V^{\prime}\frac{\partial^2 d}{\partial m_t^2} - r^{\prime\prime}}\, e^{*\prime} - \frac{\delta V^{\prime\prime}\frac{\partial d}{\partial m_t}\frac{dq_{t+1}}{dq_t}}{\delta V^{\prime}\frac{\partial^2 d}{\partial m_t^2} - r^{\prime\prime}}\, .$$

Let $\alpha = \delta V^{\prime}\frac{\partial^2 d}{\partial e_t \partial m_t}$, $\beta = \delta V^{\prime}\frac{\partial^2 d}{\partial e_t^2} - c^{\prime\prime}$, $\xi = \delta V^{\prime\prime}\frac{\partial d}{\partial e_t}\frac{dq_{t+1}}{dq_t}$, and $\rho = \delta V^{\prime}\frac{\partial^2 d}{\partial m_t^2} - r^{\prime\prime}$.

Substituting into the equations and solving for $e^{*\prime}$ and $m^{*\prime}$ we get

(E) $$e^{*\prime} = \frac{(\alpha\frac{\partial d}{\partial m_t} - \rho\frac{\partial d}{\partial e_t})\xi}{\beta\rho - \alpha^2}$$

(F) $$m^{*\prime} = \frac{\left(\alpha\frac{\partial d}{\partial e_t} - \rho\frac{\partial d}{\partial m_t}\right)\xi}{\beta\rho - \alpha^2}.$$

The denominator in (E) and (F) can be rewritten as

$$\beta\rho - \alpha^2 = \left(\delta V'\frac{\partial^2 d}{\partial e_t^2} - c''\right)\left(\delta V'\frac{\partial^2 d}{\partial m_t^2} - r''\right) - \left(\delta V'\frac{\partial^2 d}{\partial e_t \partial m_t}\right)^2 =$$
$$\left(\frac{\partial^2 d}{\partial e_t^2}\frac{\partial^2 d}{\partial m_t^2} - \frac{\partial^2 d}{\partial e_t \partial m_t}\right)(\delta V')^2 - \delta V'\frac{\partial^2 d}{\partial e_t^2}r'' - \delta V'\frac{\partial^2 d}{\partial m_t^2}c'' + c''r''.$$

From the concavity of d, $\frac{\partial^2 d}{\partial e_t^2}\frac{\partial^2 d}{\partial m_t^2} - \frac{\partial^2 d}{\partial e_t \partial m_t} \geq 0$. Thus $\beta\rho - \alpha^2 > 0$. Since $\alpha \geq 0$ and $\rho < 0$ the sign of $e^{*\prime}$ and $m^{*\prime}$ depend on the sign of ξ. If $\xi \leq 0$, then $e^{*\prime} \leq 0$ and $m^{*\prime} \leq 0$. But ξ cannot be nonnegative. This can be shown by contradiction. If $\xi \geq 0$, then $\frac{dq_{t+1}}{dq_t} \leq 0$. But it also implies $e^{*\prime} \geq 0$ and $m^{*\prime} \geq 0$, which from (3.7) means $\frac{dq_{t+1}}{dq_t} > 0$, a contradiction. Thus, $\xi < 0$ and $e^{*\prime} < 0$ and $m^{*\prime} < 0$. From (3.7) and the definition of ξ, $0 < \frac{dq_{t+1}}{dq_t} < 1$. (Q.E.D.)

Using Lemma 3.1, it follows easily that there exist a unique steady-state quality q^*. To see this, draw $q_{t+1} = q_t + d(e^*(q_t), m^*(q_t))$ and a 45° line in a diagram with q_t on the horizontal axis and q_{t+1} on the vertical axis. This gives the so-called phase diagram shown in figure 3.1. Note that, since $U'(0) = \infty$ and the costs of effort and maintenance are bounded from above, if $q_t = 0$, then $q_{t+1} = d(e^*(0), m^*(0)) > 0$. Not only the existence of a steady-state quality can be demonstrated using this phase diagram, but also it is easy to see that, starting from any $q_0 > 0$, quality will converge to this steady state as $t \to \infty$.

Using the phase line which plots q_{t+1} against q_t we can map q_0 into q_1 according to equation (3.6). We can transplot q_1 from the vertical axis to the horizontal axis by using the 45° line. By repeating this process for successive values of q, we see that the nature of the iteration is clear. Simply follow the arrowheads from q_0 bouncing between the phase line and the 45° line. This moves the value for q closer and closer to q^* while never exceeding q^*. The same analysis can be used for any $q_0 \geq q^*$.

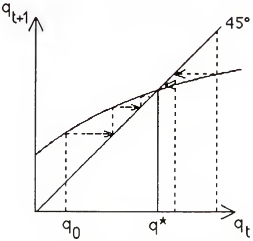


Figure 3.1. Phase diagram.

Assuming we are at the steady-state quality, equation (3.5) can be rewritten as

$$V'(q^*) = \frac{1}{1-\delta} U'(q^*). \tag{3.5a}$$

Substituting (3.5a) into (3.4a) and (3.4b) gives the first-order conditions for the Pareto-efficient effort and maintenance, e^* and m^*. These can be written as

$$\frac{\delta}{1-\delta} U'(q^*) \frac{\partial d(e^*,m^*)}{\partial e} = c'(e^*) \tag{3.4a'}$$

$$\frac{\delta}{1-\delta} U'(q^*) \frac{\partial d(e^*,m^*)}{\partial m} = r'(m^*). \tag{3.4b'}$$

This can be illustrated in figure 3.2. To get the slope of the iso-cost curve, let $C = h(e_t) + r(m_t)$ and differentiate to get

$$\frac{dm_t}{de_t} = - \frac{c'(e_t)}{r'(m_t)}.$$

The first-order conditions (3.4a′) and (3.4b′) imply that at q*, e*, and m* must satisfy

$$\frac{\dfrac{\partial d(e^*,m^*)}{\partial e_t}}{\dfrac{\partial d(e^*,m^*)}{\partial m_t}} = \frac{c'(e_t)}{r'(m_t)}.$$

That is, the slope of the iso-utility curve (the left-hand side), found by differentiating $q^* = q^* + d(e^*,m^*) = 0$, just equals the slope of the iso-cost curve. This occurs at point A in figure 3.2.

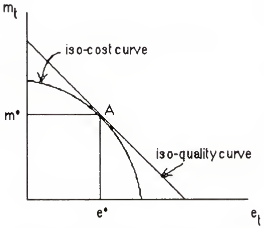


Figure 3.2. Iso-cost and iso-quality.

3.5: The Time-Dependent Supergame

Before examining equilibria in the time-dependent supergame it is necessary to specify explicitly the assumptions regarding the informational structure of the model. The actual effort and maintenance choices made cannot be verified in the present period or in any past period. This means that effort and maintenance are not contractable. However, I make the assumption that the history of quality through the present period is perfectly and costlessly verifiable by the courts. Thus, an equilibrium contract can depend on the history of quality. Note that, since the history of quality is perfectly verifiable and nature does not affect quality, each can infer what the other has chosen in each of the previous periods. Each can determine what the other has chosen because they can observe quality and know, of course, their own past choices, as well as the damage function. But remember that the landlord and tenant cannot infer the choice of effort or maintenance in the current period, because this effort and maintenance does not affect the current or past quality.

The circumstances under which the relationship between the landlord and tenant will not continue must be structured as well. Although I will not assume the existence of transactions costs for the tenant to move or the landlord to evict the tenant, I will take the reservation utilty of each to be zero. The effect is that any equilibrium contract must provide a nonnegative discounted stream of future utility from each period forward. These "participation" constraints will be defined mathematically below. There is no need to consider an extension to positive transactions costs, since it should make the constraints for participation less binding.

It is not difficult to show that the steady-state quality introduced in the previous section cannot be maintained with efficient effort and maintenance, using a continuous rental function, which depends on current quality, and without the use of "trigger" strategies, which will be introduced in the next section. If tenants and landlords do not use strategies which consider past behavior and quality, then any rental contract giving one an incentive to behave efficiently gives the other an incentive to "shirk" in either effort for the tenant, or maintenance for the landlord. This corresponds to the standard inefficiency result in double moral hazard models.

Since past quality is perfectly verifiable by the courts, it is no problem to assume that the tenant and landlord agree to a rental contract of the following form:

$$(3.8) \qquad R(h_t) = \begin{cases} R_1 & \text{if } h_t \geqq h_t^* \\ R_2 & \text{if } h_t \leq h_t^* \end{cases}$$, where R_1 and R_2 are constants with

$R_1 > R_2$ and $h_t^* = (q_0, q_1^*, q_2^*, \ldots, q_t^*)$, the Pareto efficient path for quality with q_0 as the initial quality, and $h_t = (q_0, q_1, q_2, \ldots, q_t)$. The vector inequality notation is borrowed from Takayama (1985) and $h_t \geqq h_t^*$ means $q_i \geq q_i^*$, $i = 1,2,\ldots,t$, and $h_t \leq h_t^*$ means $q_i < q_i^*$ for some $i = 1,2, \ldots ,t$. Note that it is not unrealistic to assume that the rental rate depends on past history of quality, especially the quality at the end of the last period.

A Nash equilibrium in the infinite horizon time-dependent supergame must satisfy the following:

The tenant solves

$$(3.9) \qquad \max_{e_t} \left\{ \sum_{t=0}^{\infty} \delta^t (U(q_t) - c(e_t) - R(h_t)) \right\}$$

subject to $q_{t+1} = q_t + d(e_t, m_t)$, $t = 0,1,2, \ldots, \infty$, and q_0 given.

The landlord solves

$$(3.10) \qquad \max_{m_t} \left\{ \sum_{t=0}^{\infty} \delta^t (R(h_t) - r(m_t)) \right\}$$

subject to $q_{t+1} = q_t + d(e_t, m_t)$, $t = 0,1,2, \ldots, \infty$, and q_0 given.

Of course, the constraints necessary for the participation of the landlord and tenant must also be satisfied in any equilibrium. Taking the reservation utility of each to be zero, the participation constraints for any period can be written as

$$(3.11) \qquad \sum_{t=i}^{\infty} \delta^t (U(\hat{q}_t) - c(\hat{e}_t) - R(\hat{h}_t)) \geq 0, \; i = 0,1,2, \ldots, \infty$$

$$(3.12) \qquad \sum_{t=i}^{\infty} \delta^t(R(\hat{h}_t) - r(\hat{m}_t)) \geq 0\,, \quad i = 0,1,2,\ldots,\infty$$

where $\hat{e}_t$, $\hat{m}_t$, and $\hat{q}_t$ are equilibrium values for effort, maintenance, and quality and $\hat{h}_t = (\hat{q}_0,\hat{q}_1,\hat{q}_2,\ldots)$, with $q_0 = \hat{q}_0$. This means that, in every period, an equilibrium contract $R(h_t)$ must give the landlord and the tenant a discounted stream of future utility which is nonnegative, which is at least as great as each would get otherwise.

Let us begin by examining a subgame perfect equilibrium of the game. In particular, let us examine an equilibrium when $q_0 = \hat{q}_0 < q^*$. It is useful to characterize this equilibrium, which I shall call the "grim" equilibrium, in order to facilitate the derivation of "trigger" strategy equilibrium in the next section. Assuming the participation constraints continue to be satisfied, the landlord will have a dominant strategy. Because increasing maintenance does not increase the landlord's utility now or in the future, but does decrease his utility now, the landlord will choose $\hat{m}_t = 0$ for all t. Intuitively, if the tenant will remain in the unit forever, with the constant rent there is no increase or decrease in future rent from performing maintenance. This makes it easy to solve the problem faced by the tenant, since his behavior does not affect the optimal strategy choice of the landlord. The problem can now be viewed as a dynamic optimization problem where the tenant solves (3.9) with $\hat{m}_t = 0$ for all t. This problem again allows the use of standard dynamic programming techniques.

Bellman's equation can be written as

$$(3.13) \qquad V_T(q_t) = \max_{e_t} \left\{(U(q_t) - c(e_t) - R_2) + \delta V_T(q_{t+1})\right\}$$

$$\text{where } q_{t+1} = q_t + d(e_t\,, 0) \text{ and } \hat{q}_0 < q^* \text{ given.}$$

We make the assumption that $V_T(q_t) \geq 0$ for all relevant q_t. This is simply to insure that the tenant won't leave the unit. I will address exactly what q_t are relevant in Section 3.5. The first-order condition for the right-hand side maximization problem is

$$(3.14) \qquad - c'(e_t) + \delta\, V_T'(q_t + d(e_t,0))\frac{\partial d(e_t,0)}{\partial e_t} = 0$$

and the equation of Benveniste and Scheinkman is

$$(3.15) \qquad V_T'(q_t) = U'(q_t) + \delta\, V_T'(q_{t+1}).$$

Again, there exists a unique strictly concave V_T satisfying the functional equation (3.13). (See Appendix.) Further, we know there exists a unique and time invariant optimal policy function $\hat{e}(q_t)$. (See Appendix.) Substituting this optimal policy function into the transition equation determines the optimal path of the state variable, $\hat{q}_{t+1} = \hat{q}_t + d(\hat{e}(q_t)\,,\,0)$. The derivative of this optimal transition equation can help determine whether or not there exists a unique steady-state in this equilibrium. This derivative is

$$(3.16) \qquad \frac{d\hat{q}_{t+1}}{d\hat{q}_t} = 1 + \frac{\partial d(\hat{e},0)}{\partial e}\,\hat{e}'(q_t)\,.$$

Using an approach similar to that in the previous section, I can show

$$0 \le \frac{d\hat{q}_{t+1}}{d\hat{q}_t} < 1.$$

Lemma 3.2. $0 \le \frac{d\hat{q}_{t+1}}{d\hat{q}_t} < 1.$

Proof. $0 \le \frac{d\hat{q}_{t+1}}{d\hat{q}_t}$ by contradiction.

Assume $\hat{q}_t^1 \ge \hat{q}_t^2$ and $\hat{q}_{t+1}^1 < \hat{q}_{t+1}^2$. From V_T continuous and strictly concave we have $V_T'(\hat{q}_{t+1}^1) > V_T'(\hat{q}_{t+1}^2)$. But this implies

$$\frac{c'(\hat{e}(\hat{q}_t^1))}{\delta\left(\frac{\partial d(\hat{e}(\hat{q}_t^1),0)}{\partial e_t}\right)} > \frac{c'(\hat{e}(\hat{q}_t^2))}{\delta\left(\frac{\partial d(\hat{e}(\hat{q}_t^2),0)}{\partial e_t}\right)}$$

. From the strict convexity of c(e) and the concavity of d, the above inequality implies $\hat{e}(\hat{q}_t^1) > \hat{e}(\hat{q}_t^2)$ and $\hat{q}_{t+1}^1 > \hat{q}_{t+1}^2$, a contradiction.

$\frac{d\hat{q}_{t+1}}{d\hat{q}_t} < 1$ by direct proof.

Differentiating the FOC (3.14) gives

$$- c''\hat{e}'(q_t) + \delta V_T'\frac{\partial^2 d}{\partial e_t^2}\hat{e}'(q_t) + \delta V_T''\frac{\partial d}{\partial e_t}\left[1 + \frac{\partial d}{\partial e_t}\hat{e}'(q_t)\right] = 0.$$

From (16)

$1 + \frac{\partial d}{\partial e_t}\hat{e}'(q_t) = \frac{d\hat{q}_{t+1}}{d\hat{q}_t}$. Rearranging to solve for $\hat{e}'(q_t)$ gives

$$\hat{e}'(q_t) = \frac{-\delta V_T''\frac{\partial d}{\partial e_t}\frac{d\hat{q}_{t+1}}{d\hat{q}_t}}{\delta V_T'\frac{\partial^2 d}{\partial e_t^2} - c''}$$

. Since we know $V_T' > 0$, $V_T'' < 0$, and $0 < \delta < 1$, $\hat{e}'(q_t) < 0$. Thus, from (3.16), $\frac{d\hat{q}_{t+1}}{d\hat{q}_t} < 1$. (Q.E.D.)

Using Lemma 3.2 and a phase diagram, as in figure 3.16, it is easy to verify that there exists a unique $\hat{q}$ such that $\hat{q} = \hat{q} + d(\hat{e}(\hat{q}),0)$ for any $\hat{q}_0$. We also get a little more information from the result that $\hat{e}'(q_t) < 0$. If $\hat{q}_0 > \hat{q}$, then as time passes quality must be decreasing as effort increases. The interpretation of this is that if, $\hat{q}_0 > \hat{q}$, the tenant lets $\hat{e}(q_0)$ be relatively small with $d < 0$. That is, he lets quality decline. But as quality declines he increases his effort to make quality fall by less and less each period. Of course, as $t \to \infty$, $\hat{e}(q_t) \to \hat{e}(\hat{q})$ since

$q_t \rightarrow \hat{q}$. If $\hat{q}_0 < \hat{q}$, the tenant initially puts forth great effort to cause quality to rise, and then reduces effort to allow quality to rise by less each period.

3.6: Trigger Strategy Equilibria

In this section, I will show that the first-best is attainable as a subgame perfect Nash equilibrium in this infinite-horizon time dependent supergame for δ sufficiently close to one. This is possible if the tenant and landlord use what are called "trigger strategies." The notion of trigger strategy equilibrium depends on the inability of each player in the game to deviate from perhaps efficient behavior and increase his discounted payoff. This is simply the requirement that the trigger strategies are a noncooperative equilibrium. The idea is that the landlord and tenant play the first-best levels of maintenance and effort unless in some past period the other "shirks" by letting maintenance or effort fall below the first-best levels. If shirking occurs they revert to playing strategies which end up making each of them worse off. Of course, any such "grim" reversion must be a subgame perfect equilibrium in order to be credible when used as threat. A subgame is defined as the play of the game from any given time period t onward, together with the history of the game at time t. By specifying a history, one then specifies a subgame. A set of closed-loop strategies form a subgame perfect equilibrium if the strategies form a closed-loop equilibrium in every possible subgame. I will use the subgame perfect "grim" equilibrium described in the previous section as the threats to support the first-best in equilibrium.

Assume $q_0 = q^*$. Suppose that, in any period, either the tenant or landlord deviates from the first-best levels, e^* and m^*, respectively. The immediate implication is that quality falls below q^*. If $\hat{q}_t < q^*$ is the quality level immediately following the deviation, let $\hat{e}_{t+i}$ and $\hat{m}_{t+i} = 0$, $i = 1,2,\ldots,\infty$ be the sequence of effort and maintenance which form the subgame perfect equilibrium described in the previous section (i.e. solves the game described by (3.8)-(3.12)). The values for $\hat{e}_{t+i}$ above, and the utility to the tenant, depend on the observed quality after a deviation has occurred.

Proposition 3.1 says, assuming $q_0 = q^*$, it is possible to use the equilibrium choices described above to support the efficient levels of effort and maintenance in a subgame perfect equilibrium. In order to prove Proposition 3.1 it is not necessary to solve for optimal deviations by the landlord and tenant. Instead, we can simply find an upper bound on the maximum payoff each could obtain following any possible deviation. The landlord's upper bound on the maximum payoff following any deviation is easy to solve for, since his payoff is independent of the quality following a deviation. The tenant's upper bound on the maximum payoff following any deviation is more difficult to solve for, since his maximum payoff depends on whether the quality level following a deviation is greater than, equal to, or less than $\hat{q}$. If this quality is less than or equal to $\hat{q}$ the maximum payoff is easy to describe, but when it is greater than $\hat{q}$ it is much more difficult. In either case, I shall prove that both the landlord and tenant obtain strictly lower discounted utility following any deviation, if δ is sufficiently close to one. To complete the proof, I then verify that the participation constraints may continue to be satisfied in such an equilibrium.

Proposition 3.1. There exists a rental contract of the form in (3.8) such that, for $q_0 = q^*$, the strategies

$$m_t = \begin{cases} m^* & \text{if } h_t \geq h^* \\ 0 & \text{if } h_t < h^* \end{cases} \qquad e_t = \begin{cases} e^* & \text{if } h_t \geq h^* \\ \hat{e}_t(q_t) & \text{if } h_t < h^* \end{cases}$$

$t = 0,1,2,\ldots,\infty$ is a subgame perfect equilibrium of the game for δ sufficiently close to one.

Proof. We have assumed $q_0=q^*$, implying $R(h_0)=R_1$. Without loss of generality, I shall examine all possible deviations which can occur in period 0. There are three reasonable (it never pays either to deviate by choosing effort or maintenance greater than the efficient levels) possibilities involving deviations: 1) the landlord deviates by setting $m_0' < m^*$ and $e_0 = e^*$; 2) the tenant deviates by setting $e_0' < e^*$ and $m_0 = m^*$; and 3) both deviate by setting $m_0' < m^*$ and $e_0' < e^*$. Any of these three possibilities imply $q_1' < q^*$, where $q_1' = q^* + d(e^*,m_0')$, $q_1' = q^* + d(e_0',m^*)$, or $q_1' = q^* + d(e_0',m_0')$. There are two possibilities for q_1': (i) $q_1' \leq \hat{q}$ or (ii) $q_1' > \hat{q}$, where $\hat{q}$ is the unique steady-state which solves $\hat{q} = \hat{q} + d(\hat{e}(\hat{q}),0)$.

Case (i)

If only the landlord deviates in period 0 the optimal deviation is obviously $m_0' = 0 < m^*$. The resulting maximum discounted utility the landlord can obtain is

$$L' \equiv R_1 + \frac{\delta}{1-\delta} R_2.$$

This is strictly greater than the utility he would receive had he not deviated and the tenant did. If only the tenant deviates in period 0, the upper bound on the maximum discounted utility he can obtain is

$$T' \equiv U(q^*) - R_1 + \frac{\delta}{1-\delta}\hat{U}\ , \quad \text{where } \hat{U} \equiv U(\hat{q}) - c(\hat{e}) - R_2.$$

This is just the discounted utility which would result from $e_0' = 0 < e^*$ and $q_1' = q^*$, since $e_0' > 0$ and/or $q_1' < \hat{q}$ would result in strictly lower discounted utility. Note that if they both deviated, the upper bound on the maximum discounted utility would be the same as above. The landlord will receive

$$L^* \equiv R_1 - r(m^*) + \frac{\delta}{1-\delta}(R_1 - r(m^*))$$

if neither ever deviates from first-best. The tenant receives

$$T^* \equiv U^* + \frac{\delta}{1-\delta} U^* \quad \text{, where } U^* \equiv U(q^*) - h(e^*) - R_1$$

along the first-best path. Comparing L^* and L' we find

$$L^* > L' \text{ if } \frac{\delta}{1-\delta}(R_1 - r(m^*) - R_2) > r(m^*).$$

Comparing T^* and T' we find

$$T^* > T' \text{ if } \frac{\delta}{1-\delta}(U^* - \hat{U}) > c(e^*).$$

Thus, if $U^* - \hat{U} > 0$ and $R_1 - R_2 > r(m^*)$, then as $\delta \to 1$, $L^* > L'$ and $T^* > T'$. Remember, this covers all possible deviations which could occur. To summarize case (i) $q_1' \leq \hat{q}$; when

(a) $r(m^*) < R_1 - R_2 < U(q^*) - c(e^*) - [U(\hat{q}) - c(\hat{e})]$

both the tenant and landlord are strictly worse off from any deviation from e^* and m^* for δ sufficiently close to 1.

Case (ii)

The landlord side is exactly the same as in Case (i). Following any deviation in period 0, if $R_1 - R_2 > r(m^*)$, as $\delta \to 1$ the landlords discounted utility is strictly lower as a result.

The tenant side is much more difficult. Remember the result from the previous section that $\hat{q}_t \to \hat{q}$ and $\hat{e}_t \to \hat{e}$, as $t \to \infty$, and if $q_1' > \hat{q}$, $\hat{e}_1 < \hat{e}$. This means there exists an integer N such that for any $\gamma > 0$, $n \geq N$ implies $\| (\hat{q}_n, \hat{e}_n) - (\hat{q}, \hat{e}) \| < \gamma$, where $\| \ \|$ is the Euclidean norm. Furthermore, from the continuity of the utility function we know

$$\lim_{(\hat{q}_t, \hat{e}_t) \to (\hat{q}, \hat{e})} (U(\hat{q}_t) - h(\hat{e}_t) - R_2) = U(\hat{q}) - c(\hat{e}) - R_2 = \hat{U}.$$

This means that for every $\varepsilon > 0$, there exists a γ such that $(U(\hat{q}_t) - c(\hat{e}_t) - R_2) - \hat{U}$. $< \varepsilon$ for all points for which $\| (\hat{q}_t,\hat{e}_t)-(\hat{q},\hat{e}) \| < \gamma$. Let's choose $\varepsilon = \hat{\varepsilon} > 0$. From the above, we know there exists $\gamma = \hat{\gamma}$ such that $\Delta_t \equiv (U(\hat{q}_t) - c(\hat{e}_t) - R_2) - \hat{U} < \hat{\varepsilon}$ for all points for which $\| (\hat{q}_t,\hat{e}_t)-(\hat{q},\hat{e}) \| < \hat{\gamma}$. Now let $N = \hat{N}$ be such that $\| (\hat{q}_t,\hat{e}_t)-(\hat{q},\hat{e}) \| < \hat{\gamma}$. Thus, for all $n \geq \hat{N}$, $\Delta_t < \hat{\varepsilon}$. Since $q_1' > \hat{q}$ we know $e_1 < \hat{e}$ and the greatest utility the tenant can receive following any deviation is

$$U(q_1) - c(e_1) - R_1 \equiv \hat{U}_1 > \hat{U}.$$

Also, the maximum the tenant can increase his utility in period zero by deviating is $c(e^*)$. The tenant can earn $\hat{U}_1$ until period $\hat{N} - 1$. From $\hat{N}$ on the most he can earn is $\hat{U} + \hat{\varepsilon}$. The upper bound on the maximum discounted utility to a tenant from deviating in period 0 is then

$$T'' \equiv U^* + h(e^*) + \delta\hat{U}_1 + \delta^2\hat{U}_1 + \ldots + \delta^{\hat{N}-1}\hat{U}_1 + \frac{\delta^{\hat{N}} - 1}{1 - \delta}[\hat{U} + \hat{\varepsilon}].$$

The discounted utility the tenant can earn without any deviation can be written as

$$T^* \equiv U^* + \delta U^* + \delta^2 U^* + \ldots + \delta^{\hat{N}-1}U^* + \frac{\delta^{\hat{N}} - 1}{1 - \delta}U^*.$$

Comparing T^* and T'', we see that $T^* > T''$ if

$$\frac{\delta^{\hat{N}} - 1}{1 - \delta}\left(U^* - \hat{U} - \hat{\varepsilon}\right) > c(e^*) + \delta(\hat{U}_1 - U^*) + \delta^2(\hat{U}_1 - U^*) + \ldots + \delta^{\hat{N}-1}(\hat{U}_1 - U^*).$$

Now, if $U^* - \hat{U} - \hat{\varepsilon} > 0$, then as $\delta \to 1$ the left-hand side $\frac{\delta^{\hat{N}} - 1}{1 - \delta}\left(U^* - \hat{U} - \hat{\varepsilon}\right) \to \infty$. The right-hand side is bounded above. Thus $T^* > T''$ for δ sufficiently close to 1. $U^* - \hat{U} - \hat{\varepsilon} > 0$ can be rewritten as $R_1 - R_2 < U(q^*) - c(e^*) - U(\hat{q}) + c(\hat{e}) - \hat{\varepsilon}$. Thus, if

(b) $r(m^*) < R_1 - R_2 < U(q^*) - c(e^*) - U(\hat{q}) + c(\hat{e}) - \hat{\varepsilon}$ then $L^* > L''$ and $T^* > T''$.

Also note that if (b) is satisfied (a) is also satisfied, so $T^* > T'$ as well. So (b) is the condition which must be satisfied following any deviation for both the tenant and landlord to be strictly worse off than they are along the first best path. From (b) we can see that it will be possible to find an R_1 and R_2 to satisfy this condition if $0 < U(q^*) - c(e^*) - r(m^*) - [U(\hat{q}) - c(\hat{e})] - \hat{\varepsilon}$.

Let $s^* \equiv U(q^*) - c(e^*) - r(m^*)$. This is the unique Pareto optimal per period surplus which solves the first-best problem with $q_0 = q^*$. Let $\hat{s} \equiv U(\hat{q}) - c(\hat{e})$. This is the steady-state per period surplus in the grim equilibrium with $q_0 < q^*$. Since s^* is the unique Pareto optimal, we know $s^* - \hat{s} \geq \bar{\varepsilon}$, $\bar{\varepsilon} > 0$. Now, since $\hat{\varepsilon}$ is arbitrary, choose $\hat{\varepsilon} < \bar{\varepsilon}$. Thus, (b) is satisfied.

Now it must be demonstrated that choosing R_1 and R_2 to satisfy (b) is not inconsistent with the individual participation constraints (3.11) and (3.12). If $q_1' > \hat{q}$, then $R_1 - r(m^*) > 0$, $U(q^*) - c(e^*) - R_1 > 0$, $U(\hat{q}) - c(\hat{e}) - R_2 \geq 0$, and $R_2 > 0$ are sufficient for (11) and (12) to be satisfied. Since if we combine these inequalities we simply get (b), they do not imply stricter restrictions on $R_1 - R_2$. Now we must show that the restrictions on $R_1 - R_2$ implied by the new inequalities can be satisfied. Since s^* is strictly positive, $R_1 - r(m^*) > 0$ and $U(q^*) - c(e^*) - R_1 > 0$ can be satisfied by the choice of R_1. In section 3.3, the assumption that $V_T(q_t) \geq 0$ is satisfied for all relevant q_t is sufficient to insure that $U(\hat{q}) - c(\hat{e}) - R_2 \geq 0$ and $R_2 > 0$ can be satisfied by an appropriate choice of R_2. The relevant q_t are those which will obtain following a deviation. If $q_1' = \hat{q}$, these inequalities are sufficient to satisfy the participation constraints and again pose no inconsistencies with (b). However, if $q_1' < \hat{q}$ these inequalities may not be sufficient for (3.11) and (3.12) to be satisfied. We can replace $U(\hat{q}) - c(\hat{e}) - R_2$ with $U(\hat{q}_1) - c(\hat{e}_1) - R_2 > 0$ to obtain this result. However, although this would be sufficient, it is not necessary. The weakest sufficient condition requires only $U(\hat{q}_1) - c(\hat{e}_1) > 0$, since R_2 can be zero. Regardless, making the necessary restrictions such that (11) and (12) are satisfied do not affect the condition (b) and are not inconsistent. (Q.E.D.)

3.7: Multiple Equilibria, Renegotiation, and Finite Horizon

The equilibrium described in Proposition 3.1 is only one of many. There are many other equilibria of the same type. For instance, suppose that $q_0 = q^*$ and $e_t = e^* - \varepsilon$ and $m_t = m^* + \zeta$, $t=0,1,2,\ldots,\infty$, where $d(e^*-\varepsilon, m^*+\zeta)=0$ and $\varepsilon,\zeta>0$. That is, suppose q^* is supported with the tenant putting forth a little less than efficient effort, and the slack is taken up by the landlord putting forth a little more than efficient maintenance. In this case, the landlord will receive $R_1 - r(m^*+\zeta)$ and the tenant will receive $U(q^*) - c(e^*-\varepsilon) - R_1$. Condition (a) becomes

$$r(m^*+\zeta) < R_1 - R_2 < U(q^*) - c(e^*-\varepsilon) - [U(\hat{q}) - h(\hat{e})].$$

Case (ii) can also be adjusted in a similar way. The tenant receives higher utility each period than if he chooses e^*, so he will be worse off following any deviation when the "grim equilibrium" is played in response and R_1 and R_2 chosen appropriately. If the landlord receives a payoff in the first-best case which is strictly greater than zero, then R_1 and R_2 can be chosen such that the above strategies form a subgame perfect Nash equilibrium for δ sufficiently close to one. There are obviously a continuum of such equilibria. Clearly, it is also possible to show that the strategies $e^*+\omega$ and $m^*-\upsilon$ can be sustained as a subgame perfect Nash equilibrium through the use of trigger strategies, if the tenant receives strictly positive surplus from the first-best choices. Again, there are a continuum of such equilibria. These equilibria are illustrated in figure 3.3. The darkened segment of the iso-quality curve for q^* illustrates examples of choices of effort and maintenance which can be supported as a subgame perfect Nash equilibrium.

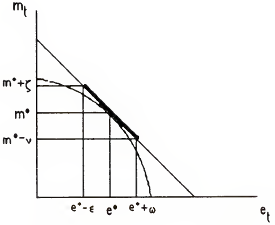


Figure 3.3. Multiple equilibria.

Equilibria with d=0 each period are not likely to be the only ones that yield higher surpluses than the grim equilibrium. The implication is that contracts and strategies yielding strictly greater surpluses than the grim equilibrium may also be supported as subgame perfect Nash equilibria. Analysis of these other equilibria are beyond the scope of this dissertation.

Throughout this chapter, I have assumed that renegotiation is not possible. Some current research has focused on supergames which account for the possibility of renegotiation. This literature suggests that the ability to prevent deviations would be undermined if renegotiation was permitted. In Proposition 3.1, I showed that R_1 and R_2 are chosen such that both landlord and tenant are strictly worse off following any deviation. This is possible because the size of the combined surplus must decrease following a deviation. Clearly they will eventually have an incentive to rescind the original contract and agree to a new one which can increase the combined surplus and make both better off. For instance, they may write a contract which allows the return to R_1 if quality is returned to q^*. If the tenant knows that the landlord will always want to renegotiate following a deviation, then nothing will prevent the tenant from deviating. If renegotiation cannot be prevented, the condition that there are no ex post Pareto improvements from recontracting must be imposed as a constraint in the original contract. Clearly, this is an avenue which deserves further research in the context of this model.

In a finite, T-period, framework, assuming the landlord removes the unit from the market after T periods (and does not care about quality at time T), the first-best cannot be obtained by using a rental contract of the form in (3.8). To see this, note that the tenant will always deviate from e^* in some period $t \leq T$. The landlord should be able to anticipate this optimal deviation and thus set $m_t = 0$. The usual backward induction argument would seem to indicate that the only subgame perfect equilibrium is one which resembles the grim equilibrium. However, since assuming the landlord does not care about quality at time T is difficult to imagine and we cannot appeal to steady-state properties, I shall not pursue this further. The more realistic case where the landlord keeps the unit on the market after T periods should be incorporated into any finite-horizon model.

In finite-horizon models, where the landlord keeps the unit after T periods, the analysis becomes more complicated. Some problems which must be considered are the cost of renovation and marketing vacant units, the probability of rerenting the unit, and structure of the market. For instance, if the tenant will always leave after one period, no contract can solve the tenant moral hazard problem. The tenant will always put forth zero effort. What the landlord does depends on the future possibilities and costs of rerenting. It does seem obvious that the landlords optimal behavior will not be to perform zero maintenance. The reason is that the future rent and probability of renting depend on quality. Also, it is more costly to renovate the unit to improve quality than to perform regular maintenance. The problem essentially becomes one of determining the optimal maintenance policy for the landlord. Arnott et al (1983) and Vorst (1987), for instance, examine the optimal maintenance policy for the landlord. Vorst incorporates the tenant externality by examining maintenance where the tenant behavior contributes to some stochastic influence on quality. Guasch and Marshall (1987) develop a model, to explain tenure discounts, which can offer assistance in incorporating costs of renovation and marketing vacant units in a model of rental housing.

One possible avenue for further research would consider equilibria in a finite-horizon model where the observable quality extends to the end of the current period. The contract could involve the use of damage deposits. Clearly the finite-horizon will not allow for Pareto-efficiency. However, it is interesting to consider the equilibria which can occur and the role of the damage deposit in these equilibria. These issues will be examined in Chapter 4 of this dissertation.

Although Proposition 3.1 requires an unbounded horizon, the results continue to hold with the reasonable assumption that there is a constant probability x in (0,1) that the relationship will continue (assuming also that the landlord removes the unit from the market if the tenant does not continue to rent). For example, x can be interpreted as the probability that the tenant will continue to rent this unit. The result is that the relationship will end at some point in time with probability 1. The date the relationship will actually end, however, is not known in advance. The relevant discount factor then becomes $\underline{\delta} = \delta x$ and Proposition 3.1 is simply adjusted accordingly. That is, for δ and x sufficiently close to one, the first-best effort and maintenance can be obtained. Again, it seems more realistic to assume the landlord can rerent (or sell) the unit if the relationship ends. In this case, one would expect that, given an constant exogenous probability of rerenting, a similar result can be obtained. The discount factor would depend on x and the probability of rerenting. For a high enough probability of rerenting, the first-best effort and maintenance can be obtained for δ and x sufficiently close to one.

3.8: Conclusions

In this chapter, I developed a model of rental housing with an infinite-horizon which illustrates that it is possible to maintain efficient sharing of effort and maintenance to sustain quality at the efficient level. This was possible through the use of "trigger" strategies, where each behaves efficiently unless, sometime in the past, one "deviated" by reducing effort or maintenance. Of course, the Pareto-efficient solution is not the only equilibrium of the game. There exists many equilibria. However, it not uncommon to assume the agents focus on the socially efficient equilibrium.

Besides the avenues for further research addressed above, it is obvious that it would be appealing to add a stochastic component to the damage function. It also may not be difficult to extend the results of this chapter to include the epsilon-equilibria Radner developed for finite-horizon oligopoly models. I also believe that this model and its results can be generalized with a continuous rental function. It is fairly easy to prove the existence of an open-loop "grim" equilibrium, however, the strategies in such an equilibrium would not, in general, be credible when used as trigger strategies. It is much more difficult to show the existence and properties of closed-loop equilibrium which could lead us to derive subgame perfect equilibria in the time-dependent supergame.

CHAPTER 4
THE ROLE OF DAMAGE DEPOSITS IN RENTAL HOUSING MARKETS

4.1: Introduction

In this chapter, I examine the role of damage deposits in a competitive rental housing market. In markets where the tenant will occupy a unit a known finite number of periods, we already know (from Section 3.7) the tenant will choose less-than-efficient effort to maintain and care for the unit. This is a result of the assumption that tenant effort is not contractible. This is the problem of tenant moral hazard.

To illustrate this tenant moral hazard, suppose that the tenant stays in a unit for exactly one period. The beginning-of-the-period quality is perfectly and costlessly verifiable to everyone. However, something prevents contracting on the end-of-period quality. One possible reason has all transfers taking place at the beginning of the period. Thus, we effectively eliminate the possibility of contracting on the end-of-period quality even if it becomes verifiable. The obvious result is that the tenant will choose zero effort. This is because there is no way to punish the tenant for inefficient choices.

If we allow for transfers to occur at the end of the period, we can solve this problem by using damage deposit requirements. The tenant pays a damage deposit, in addition to rent, at the beginning of the period. If, at the end of the period, quality falls below some threshold, the deposit is kept by the landlord. Otherwise, it is returned. Obtaining efficient choices by tenants is possible because any damage which occurs can be attributed to insufficient tenant effort.

Now suppose that, even though quality at the end of the period is verifiable, damage cannot be attributed directly to tenant behavior. Nature may affect quality through damage or, as in the last chapter and this one, landlord maintenance also affects damages. The damages which may occur do not give perfect information about tenant behavior. The choice by the tenant cannot be separated from that of the landlord simply by observing the end-of-period quality. The problem, then, becomes one of double moral hazard.

In Chapter 3, it was argued that, if the tenant-landlord relationship lasts a known finite number of periods, efficient choices of both effort and maintenance will not obtain. This was because it was not possible to separate effort and maintenance by observing quality. Any balanced budget mechanism giving one an incentive to behave efficiently gives the other an incentive to behave inefficiently. Of course, in this chapter the result will be the same. In fact, if we allow for damage deposits, the landlord wants to choose inefficient maintenance to lower costs as before, but also to keep the deposit.

Although Pareto-efficient choices will not obtain in equilibrium under double moral hazard, it is still possible that damage deposits can play a useful role. Under certain conditions regarding the cost of effort relative to the cost of maintenance, equilibrium with damage deposits will result in greater efficiency than without them. However, as I will show, effort and maintenance cannot both be positive in equilibrium. If tenants put forth positive effort, and they will only if damage deposits are positive in equilibrium, then landlords perform zero maintenance. If landlords perform positive maintenance, then tenants will put forth zero effort. In the latter case, there is no role for damage deposits.

It is somewhat surprising that, in equilibrium with positive damage deposits, landlords do not perform any maintenance. In fact, even though tenant effort is too low to prevent the deterioration of quality over time, landlords do not put forth positive maintenance. With damage deposits to control the tenant moral hazard problem, one would expect the landlord to perform at least some maintenance. Of course, we know the reason for this is that the informational structure results in a problem of double moral hazard.

Perhaps the main results in this chapter concern the equilibrium path of quality and rent over time. When there is double moral hazard, quality deteriorates over time and rent falls over time. Also, per-period profits begin positive, but eventually become negative in a competitive equilibrium.

With or without positive damage deposits in equilibrium, quality will deteriorate over time. In Definition 4.1, I describe the properties of a competitive equilibrium in this model. The equilibrium requirement that tenants must be indifferent between quality levels implies that rent falls as quality falls over time. In either equilibrium, the free entry condition implies that profits over the infinite horizon must equal zero. That is, the discounted sum of rents earned over time minus the discounted sum of landlord costs must equal zero. Thus, per period profit is initially positive, but at some point in time becomes negative.

These results seem consistent with what is actually observed. Quality does deteriorate over time, even though with appropriate effort and/or maintenance it does not necessarily have to. Also, it is likely that per-period profits are initially positive, but at some point become negative.

This chapter is divided into four main sections. Of course Section 4.1, provides an introduction to this chapter. In Section 4.4, the last section, I provide some concluding remarks and ideas for further research. Each of the two sections in between are divided into five subsections. In Section 4.2, I examine a model where only the tenant can affect the path for quality. An introduction to the model and its main results are provided in Section 4.2.1. After specifying the notation and assumptions in Section 4.2.2, I examine the

Pareto-efficient paths for quality and effort in Section 4.2.3. In Section 4.2.4, I show that equilibrium without damage deposits is inefficient. In Section 4.2.5, I show that with very simple discrete damage deposits, equilibrium will obtain with Pareto-efficient choices by tenants. In Section 4.3, I extend the model to include the effect of landlord maintenance on quality. In Section 4.2.1, I introduce the model and summarize some of the results. Section 4.3.2 specifies the notation and assumptions. Pareto-efficiency is examined in Section 4.3.3. Also, in Section 4.3.3, I introduce the properties of a competitive equilibrium in this model. In Section 4.3.4, equilibrium without damage deposits is characterized. Finally, in Section 4.3.5, I demonstrate the potential for gains from using deposits and show how this depends on the costs of effort relative to maintenance. I also discuss some of the more surprising results concerning the equilibrium path of quality and rent in Section 4.3.5.

4.2: Tenant Moral Hazard

4.2.1: Introduction

In Section 4.2, I develop a model where only the tenant in rental housing markets can influence the path of quality over time. In this model, there is a tenant moral hazard problem. The tenant does not have an incentive to put forth efficient effort to maintain the unit he occupies. The reason is that the tenant stays in the unit only one period and cannot be forced to pay for any damages for which he is responsible. However, I show that if we allow for very simple damage deposits in the rental contract , an equilibrium with Pareto-efficient choices of effort will obtain.

4.2.2: Assumptions and Notation

In any period $t=1,2,\ldots,\infty$, the quality $q_t \in Q$, where $Q \equiv \{q \mid 0 \leq q \leq \infty\}$ depends on last periods quality, $q_{t-1} \in Q$ and the tenant's effort last period, $e_{t-1} \in E$, where $E \equiv \{ e \mid 0 \leq e \leq \overline{e}\}$. Effort affects next period quality through the damage function $d(e_t)$. Assume $d \in D$, where $D \equiv \{ d \mid d_{min} \leq d \leq d_{max}\}$, $d_{min} = d(0) < 0$, $d_{max} = d(\overline{e}) > 0$, $d'(e_t) > 0$, and $d''(e_t) \leq 0$. The transition equation for the state variable, quality, is

$$(4.2.1) \qquad q_t = q_{t-1} + d(e_{t-1}).$$

Every tenant in this market has an identical utility function,

$$(4.2.2) \qquad U(q_t) - c(e_t) - R_t.$$

Assume $U'(q_t) > 0$, $U''(q_t) \leq 0$, $U'(0) = \infty$, and $U'(\infty) = 0$. The cost of effort to the tenant is $c(e_t)$. Assume $c'(e_t) > 0$ and $c''(e_t) > 0$. The rent, R_t, will be discussed in more detail below.

Let the cost of building initial quality be $K(q_0)$, where $K'(q_0) > 0$ and $K''(q_0) > 0$. When financed over an infinite horizon, the user cost of the capital required to obtain initial quality q_0, can be written as $(1 - \delta)K(q_0)$, where $\delta = \frac{1}{1 + i}$ is the discount factor and i is the interest rate. This user cost of capital is the opportunity cost of using capital to provide quality. It is the cost which makes the net present value of investing $K(q_0)$ in initial quality equal to zero. Hereafter, I will refer to $(1 - \delta)K(q_0)$ as the cost of initial quality.

Landlord utility in each period t can be written as $R_t - (1 - \delta)K(q_0)$.

4.2.3: Pareto Efficiency

The Pareto-efficient initial quality and path for quality, along with the efficient path for effort can be determined by maximizing the discounted sum of tenant and landlord utility subject to the transition equation. This is a model with transferable utility. Let

(4.2.3) $W_t = U(q_t) - c(e_t) - (1 - \delta)K(q_0)$.

The objective function for the infinite-horizon Pareto-efficient problem can be written as

(4.2.4) $$\max_{(q_0, e_t)} \left\{ \sum_{t=0}^{\infty} \delta^t W_t \right\}, \; t = 0,1,2, \ldots, \infty.$$

subject to $q_{t+1} = q_t + d(e_t)$, $q_0 > 0$, and $e_t \geq 0$.

As in Chapter 3, the structure of this problem permits the use of dynamic programming techniques. Bellman's equation can be written as

(4.2.5) $$V(q_t) = \max_{(q_0, e_t)} \left\{ W_t + \delta\, V(q_{t+1}) \right\}$$

where $q_{t+1} = q_t + d(e_t)$. Using Theorem A.1, in the Appendix, the unique V satisfying (4.2.5) solves (4.2.4). Furthermore, using Theorem A.2, V is strictly concave and from Theorem A.3, we have convergence to a unique time-invariant policy function $e^*(q_t)$, where e^* is chosen to solve the right-hand-side maximization in (4.2.5). The first-order conditions for this maximization are

(4.2.6) $U'(q_t) - (1 - \delta)K'(q_0) + \delta V'(q_{t+1}) = 0$ and

(4.2.7) $-c'(e_t) + \delta V'(q_{t+1})d'(e_t) = 0$.

Applying Theorem A.4, in the Appendix, V is differentiable giving a version of the equation of Benveniste and Scheinkman

(4.2.8) $V'(q_t) = U'(q_t) + \delta V'(q_{t+1})$.

This equation will be used below to help find the efficient initial quality.

In order to characterize the Pareto-efficient solution, I shall first describe the efficient path for effort and quality given any initial quality, $q_0 > 0$. Then, to pin down the efficient initial quality, I will make a simplifying assumption.

I will show that given our optimal effort function $e^*(q_t)$, there exists a unique value, q^*, such that if $q_t = q^*$, then $q_{t+i} = q^*$ is optimal. Starting from any $q_0 > 0$ we have convergence to this unique value. I will call q^* the steady-state quality.

Substituting the optimal effort function into the transition equation, we have

(4.2.9) $\quad q_{t+1} = q_t + d(e^*(q_t))$.

Differentiating (4.2.9) with respect to q_t gives

(4.2.10) $\quad \dfrac{dq_{t+1}}{dq_t} = 1 + d'(e^*)e^{*\prime}(q_t)$.

Substituting again the optimal effort function into (4.2.7) and differentiating with respect to q_t gives

(4.2.11) $\quad -c''e^{*\prime} + \delta V'(q_{t+1})d'(e^*)e^{*\prime} + \delta V''(q_{t+1})\dfrac{dq_{t+1}}{dq_t}d' = 0.$

Rearranging (4.2.11) gives

(4.2.12) $\quad e^{*\prime} = \dfrac{-\delta V''\dfrac{dq_{t+1}}{dq_t}d'}{\delta V'd'' - c''}$.

The denominator in (4.2.12) is negative. If $\dfrac{dq_{t+1}}{dq_t} \leq 0$, then $e^{*\prime} \geq 0$, since $V'' < 0$ and $d' > 0$. But $e^{*\prime} \geq 0$ implies, from (4.2.10) $\dfrac{dq_{t+1}}{dq_t} \geq 1$, a contradiction. Thus, $\dfrac{dq_{t+1}}{dq_t} > 0$. If $\dfrac{dq_{t+1}}{dq_t} > 0$, then $e^{*\prime} < 0$ and $\dfrac{dq_{t+1}}{dq_t} < 1$.

Now that it has been established that $0 < \dfrac{dq_{t+1}}{dq_t} < 1$, we can draw a phase diagram, like the one in figure 3.1 in Chapter 3, to illustrate the convergence to q^* starting from any $q_0 > 0$. Note that at the steady-state quality level, $d(e^*) = 0$, by definition.

Now we must characterized the choice of efficient initial quality, q_0^*. It seems unreasonable to have $q_0^* < q^*$. This would mean that it is efficient to build initial quality low and let the tenant renovate the unit to increase quality each period until the steady-state level is reached. To preclude this unreasonable case, we can assume $\delta(1-\delta)K'(q^*) \leq \frac{c'(e^*)}{d'(e^*)}$. This means that, at the steady-state quality, it is less expensive to increase quality by one unit in any period by increasing initial quality than by increasing effort. The discount factor enters this inequality, because effort does not affect quality until the end of the period. For simplicity, I will assume the above holds with equality. In this case, I will show that the efficient initial quality, q_0^*, is equal to q^*. That is, efficiency calls for initial quality to be built to the steady-state level. Later in this chapter, I examine cases where the efficient initial quality differs from the steady-state quality that will result.

To show $q_0^* = q^*$, begin by substituting (4.2.8) into (4.2.6) and rearranging (4.2.6) and (4.2.7) to get

(4.2.6´) $V'(q_t) = (1-\delta)K'(q_0)$ and

(4.2.7´) $\delta V'(q_{t+1}) = \frac{c'(e_t)}{d'(e_t)}$.

In period zero, (4.2.6´) and (4.2.7´) become

(4.2.13) $V'(q_0) = (1-\delta)K'(q_0)$ and

(4.2.14) $\delta V'(q_1) = \frac{c'(e_0)}{d'(e_0)}$.

Combining (4.2.13) and (4.2.14) we have the following equation which must be satisfied:

(4.2.15) $$\frac{V'(q_0)}{V'(q_1)} = \frac{\delta(1-\delta)K'(q_0)}{\frac{c'(e_0)}{d'(e_0)}}.$$

If $q_0 > q^*$, then $K'(q_0) > K'(q^*)$. Also, if $q_0 > q^*$, we know $e_0 < e^*$ (from $e^{*\prime} < 0$). This implies $c'(e_0) < c'(e^*)$ and $d'(e_0) \geq d'(e^*)$. Since, by assumption

$\frac{\delta(1-\delta)K'(q^*)}{\frac{c'(e^*)}{d'(e^*)}} = 1$, we know $\frac{\delta(1-\delta)K'(q_0)}{\frac{c'(e_0)}{d'(e_0)}} > 1$ when $q_0 > q^*$ and, from

(4.2.15), $\frac{V'(q_0)}{V'(q_1)} > 1$. But $q_0 > q_1$ implies $\frac{V'(q_0)}{V'(q_1)} < 1$ from the strict concavity of V. This is a contradiction. Also, if $q_0 < q^*$, then $K'(q_0) < K'(q^*)$. Also, if $q_0 < q^*$, we know $e_0 > e^*$ (from $e^{*\prime} < 0$). This implies $c'(e_0) > c'(e^*)$ and $d'(e_0) \leq d'(e^*)$. Since, by assumption

$\frac{\delta(1-\delta)K'(q^*)}{\frac{c'(e^*)}{d'(e^*)}} = 1$, we know $\frac{\delta(1-\delta)K'(q_0)}{\frac{c'(e_0)}{d'(e_0)}} < 1$ when $q_0 < q^*$ and, from

(4.2.15), $\frac{V'(q_0)}{V'(q_1)} < 1$. But $q_0 < q_1$ implies $\frac{V'(q_0)}{V'(q_1)} > 1$ from the strict concavity of V. Thus, $q_0^* = q^*$.

We have completely characterized the Pareto-efficient quality and effort. Specifically, $q_t^* = q^*$ and $e_t^* = e^*$, $t = 0,1,2,\ldots,\infty$.

4.2.4: Equilibrium Without Damage Deposits

Before examining equilibrium without damage deposits, I must describe the informational structure of this model. Remember that, in this model, each tenant will occupy the unit for exactly one period. Furthermore, I assume there are no transactions costs to the tenant (or landlord) when the tenant leaves. For illustrative purposes, I assume that transfers occur only at the beginning of the period in this section. Even though the quality at the end of the period is perfectly and costlessly verifiable (everyone knows $d(\cdot)$ and the beginning of the period quality), I do not allow end-of-period transfers. By doing this, I am not allowing the end-of-period quality to be contractible. In the next section, I will allow its contractibility by permitting the use of simple damage deposits.

In each period t, the tenant will choose e_t to solve the maximization problem

$$\max_{e_t} \{U(q_t) - c(e_t) - R_t\}, \text{ given } q_t.$$

This obviously implies $e_t = 0$, $t=0,1,2,\ldots,\infty$. This is sufficient to show that equilibrium without damage deposits is Pareto-inefficient. The intuition is, of course, that the tenant cannot be punished for choosing less-than-efficient effort. This is the standard tenant moral hazard problem.

4.2.5: Equilibrium with Damage Deposits

Quality is perfectly and costlessly verifiable at the end of the period. This permits the use of quality-contingent end-of-period transfers. To do this, I will allow the rental contract to include a simple damage deposit, which may be returned at the end of the period. In addition to rent, the tenant pays a damage deposit at the beginning of the period. If, at the end of the period, quality is greater than or equal to some threshold quality, the deposit is returned to the tenant. However, if the end-of-period quality is below the threshold quality, the landlord keeps the deposit. The result, as we shall see, is that Pareto efficiency can be supported in equilibrium.

With these simple discrete damage deposit contracts, we can write the effective rent as

$$R_t + \gamma D, \text{ where } \gamma = \begin{cases} 0 & \text{if } q_{t+1} \geq q^T \\ 1 & \text{if } q_{t+1} < q^T \end{cases}.$$

The rent, R_t, and damage deposit, D, is paid at the beginning of the period. The threshold quality is q^T. If $q_{t+1} \geq q^T$, the effective transfer is R_t and if $q_{t+1} < q^T$, the effective transfer is $R_t + D$. It is assumed that D is kept in an interest-bearing account, so that the actual deposit returned to the tenant or kept by the landlord at the end of period t is not discounted in period t.

The first question which must be answered concerns whether or not there exists a deposit mechanism sufficient to induce efficient choices of effort. If $q_0 = q^*$, then any $D \geq c(e^*)$, with $q^T = q^*$, will induce the tenant to put forth exactly e^* each period. The tenant solves the maximization problem

$$\max_{e_t} \left\{U(q_t) - c(e_t) - R_t - \gamma D\right\}, \text{ subject to } q_{t+1} = q^* + d(e_t),$$
$$\text{where } \gamma = \begin{cases} 0 & \text{if } q_{t+1} \geq q^T \\ 1 & \text{if } q_{t+1} < q^T \end{cases} \text{ and } D \geq c(e^*).$$

The minimum deposit to induce efficient choices on the part of tenants is $D = c(e^*)$.

Assuming there exists some surplus each period (i.e. $U(q^*) - c(e^*) - (1-\delta)K(q^*) > 0$), without specifying the nature of competition, there is a continuum of rental rates with $D = c(e^*)$ and $q^T = q^*$ which can support Pareto efficiency. Any R such that $U(q^*) - c(e^*) \geq R \geq (1 - \delta)K(q^*)$ is a candidate for an equilibrium contract. However, with competition for tenants, the rent will be bid down so as to maximize tenant utility subject to the landlord earning nonnegative profits (utility). This implies that the contract with $R = (1 - \delta)K(q^*)$, $D = c(e^*)$, and $q^T = q^*$ will be chosen by tenants in equilibrium with $e_t = e^*$, $q_t = q^*$, and $\gamma = 0$, for $t=0,1,2,\ldots,\infty$.

Although I have not discussed fully the nature of competitive equilibrium in the context of this simple model, once the more complex model of the next section is introduced, the informal results of this model can be formalized. In fact, looking ahead, I can simply apply Proposition 4.4 to get the result that equilibrium contracts call for damage deposits to be positive to support Pareto-efficient choices of effort.

4.3: Tenant and Landlord Moral Hazard

4.3.1: Introduction

In Section 4.3, I extend the model developed in Section 4.2 to include landlord maintenance. Maintenance by the landlord will affect quality through the damage function of the transition equation. I retain the assumption from the previous section that tenants occupy units for only one period. The immediate consequence is that it will not be possible to design contracts to obtain Pareto-efficient choices of both effort and maintenance.

I will show that, even though the informational structure allows for contracting on the end-of-period quality through simple damage deposits, it is not possible to obtain Pareto-efficient choices in equilibrium. The intuition is that damage deposits which may induce efficient behavior on the part of tenants result in inefficient choices by the landlord to retain the deposit. However, under certain assumptions regarding the relative costs of effort and maintenance, it is possible to introduce damage deposits and obtain a more efficient outcome than would otherwise occur.

In Section 4.3.2, I specify the assumptions and notation relevant to the current model. In Section 4.3.3, I characterize the Pareto-efficient quality, effort, and maintenance and introduce the properties of a competitive equilibrium in this model. I then analyze equilibrium when damage deposits are not allowed in Section 4.3.4. Finally, in Section 4.3.5, I derive equilibria with damage deposits under various assumptions concerning the relative costs of effort and maintenance.

4.3.2: Assumptions and Notation

The transition equation is now written as

(4.3.1) $q_{t+1} = q_t + d(e_t, m_t)$,

where $q_t \in Q$, $e_t \in E$ (Q and E were defined in Section 4.2.2), and $m_t \in M \equiv \{ m \mid 0 \le m \le \bar{m}\}$. Assume $d(\cdot,\cdot)$ is concave with the following properties:

$$d \in D,\ D \equiv \{ d : d_{min} \le d \le d_{max} \} \subset \Re,$$

where $d_{min} \equiv d(0,0) < 0$ and $d_{max} \equiv d(\bar{e},\bar{m}) > 0$,

$$\frac{\partial d}{\partial e_t} > 0,\ \frac{\partial d}{\partial m_t} > 0,\ \frac{\partial^2 d}{\partial e_t^2} \le 0,\ \frac{\partial^2 d}{\partial m_t^2} \le 0,\ \frac{\partial^2 d}{\partial e_t \partial m_t} \ge 0,\ \text{and}$$

$$\frac{\partial^2 d}{\partial e_t^2}\frac{\partial^2 d}{\partial m_t^2} - \left(\frac{\partial^2 d}{\partial e_t \partial m_t}\right)^2 \ge 0.$$

Notice that this transition equation and its assumptions are identical to those employed in Chapter 3 of this dissertation.

Each tenant's utility function is again written as

(4.3.2) $U(q_t) - c(e_t) - R_t$.

The cost of initial quality is $(1 - \delta)K(q_0)$, where $\delta = \frac{1}{1 + i}$ is the discount factor and i is the interest rate. The assumptions regarding the utility function and cost of initial quality are identical to those in Section 4.2.2.

Landlord utility in each period can be written as $R_t - r(m_t) - (1 - \delta)K(q_0)$. The cost of maintenance is $r(m_t)$, where $r'(m_t) > 0$ and $r''(m_t) > 0$.

4.3.3: Pareto Efficiency

The Pareto-efficient quality and path for quality, along with the efficient path for effort and maintenance, can be determined by maximizing the discounted sum of tenant and landlord utility subject to the transition equation. Note that this is a problem where the utility is transferable so that the unique Pareto-efficient choices and quality can be determined in this way. Let

(4.3.3) $W_t \equiv U(q_t) - c(e_t) - r(m_t) - (1 - \delta)K(q_0)$.

Note that with our assumptions, W_t is strictly concave in $(e_0,e_1,\ldots,e_t,m_0,m_1,\ldots,m_t,q_0)$. The objective function in the infinite horizon problem can be written as

$$(4.3.4) \qquad \max_{(q_0, e_t, m_t)} \left\{ \sum_{t=0}^{\infty} \delta^t W_t \right\}$$

subject to $q_{t+1} = q_t + d(e_t, m_t)$, $t = 0,1,2, \ldots, \infty$, $q_0 > 0$, $e_t \geq 0$, and $m_t \geq 0$.

As in Chapter 3, the structure of this model allows the use of standard dynamic programming techniques. Bellman's equation can be written as

$$(4.3.5) \qquad V(q_t) = \max_{(q_0, e_t, m_t)} \left\{ W_t + \delta\, V(q_{t+1}) \right\},$$

where $q_{t+1} = q_t + d(e_t, m_t)$. Applying Theorem A.1, in the Appendix, the unique V satisfying (4.3.5) solves (4.3.4). Theorem A.2, in the Appendix, implies that V is strictly concave. Using Theorem A.3, we have convergence to time invariant effort and maintenance functions, $e^*(q_t)$ and $m^*(q_t)$, where e^* and m^*, along with the optimal q_0, are chosen to solve the right-hand-side maximization in (4.3.5). The first-order conditions for this maximization are

$$(4.3.6) \qquad U'(q_t) - (1 - \delta)K'(q_0) + \delta V(q_{t+1}) = 0,$$

$$(4.3.7) \qquad -\, c'(e_t) + \delta\, V'(q_{t+1}) \frac{\partial d(e_t, m_t)}{\partial e_t} = 0, \text{ and}$$

$$(4.3.8) \qquad -\, r'(m_t) + \delta\, V'(q_{t+1}) \frac{\partial d(e_t, m_t)}{\partial m_t} = 0.$$

Applying Theorem A.4, in the Appendix, V is differentiable giving a version of the equation of Benveniste and Scheinkman

$$(4.3.9) \qquad V'(q_t) = U'(q_t) + \delta\, V'(q_{t+1}).$$

This equation will be used later to help solve for the efficient initial quality.

In order to characterize the Pareto-efficient solution, I will first describe the efficient path for quality, effort, and maintenance for any $q_0 > 0$. I will then make a simplifying assumption to pin down the efficient q_0.

First, note that the problem of describing the Pareto-efficient path for quality, effort, and maintenance for any $q_0 > 0$ is exactly the same problem encountered in Section 3.4 of Chapter 3. In that section, I proved that quality will converge to a unique steady-state level, q^*. In fact, I need only apply Lemma 3.1 to the current problem to get the same result in this chapter. I will rewrite Lemma 3.1 below. Note that, although I use the same notation to describe Pareto-efficient choices of quality, effort, and maintenance in this section as I did in Section 4.2, they are not necessarily the same in this model and the model of Section 4.2.

Substituting the optimal policy functions into the transition equation (4.3.1) we have

$$q_{t+1} = q_t + d(e^*(q_t)\,,\, m^*(q_t)), \tag{4.3.10}$$

a first-order (perhaps) nonlinear difference equation. Note that I have left out the "*" in the quality terms along the Pareto-efficient path for notational convenience. Differentiating (4.3.10) with respect to q_t gives

$$\frac{dq_{t+1}}{dq_t} = 1 + \frac{\partial d(e^*,m^*)}{\partial e} e^{*\prime}(q_t) + \frac{\partial d(e^*,m^*)}{\partial m} m^{*\prime}(q_t). \tag{4.3.11}$$

Equations (4.3.10) and (4.3.11) are useful to understand the proof, which I will not rewrite, of Lemma 3.1.

Lemma 3.1 (from Chapter 3) states that $0 \leq \frac{dq_{t+1}}{dq_t} < 1$. Applying this result, we need only use a phase diagram, like the one employed in Chapter 3, to describe the convergence the q^*.

In the proof of Lemma 3.1, it was discovered that if initial quality is below q^*, effort and maintenance will begin high (positive) and then fall (remaining positive) each period as quality approaches q^*. Similarly, if initial quality is above q^*, effort and maintenance begin very low and then increase each period as quality approaches q^*.

Now that the path towards q^* has been summarized, we can solve for the efficient choice of initial quality, q_0^*. It seems unreasonable to have $q_0^* < q^*$, since this would imply that it is efficient to let tenants and the landlord renovate the unit to increase quality each period until the steady-state level is reached. I can rule out this case by assuming

$$\delta(1-\delta)K'(q^*) \le \frac{c'(e^*)}{\left(\frac{\partial d(e^*,m^*)}{\partial e}\right)} = \frac{r'(m^*)}{\left(\frac{\partial d(e^*,m^*)}{\partial m}\right)} .$$

However, for simplicity, I assume that this holds with equality. In this case, I will prove that the efficient initial quality is equal to q^*. That is, efficiency calls for initial quality to be built to the steady-state level. In Sections 4.3.4 and 4.3.5, the efficient initial quality will differ from the efficient steady-state quality at the best feasible contract.

To show that $q_0^* = q^*$, begin by substituting (4.3.9) into (4.3.6) and rearranging (4.3.6) – (4.3.8) to get

(4.3.6′) $V'(q_t) = (1-\delta)K'(q_0)$,

(4.3.7′) $\delta V'(q_{t+1}) = \frac{c'(e_t)}{\left(\frac{\partial d(e_t,m_t)}{\partial e_t}\right)}$, and

(4.3.8′) $\delta V'(q_{t+1}) = \frac{r'(m_t)}{\left(\frac{\partial d(e_t,m_t)}{\partial m_t}\right)}$.

Evaluating these equations at period zero gives

(4.3.6′′) $V'(q_0) = (1-\delta)K'(q_0)$,

(4.3.7′′) $\delta V'(q_1) = \frac{c'(e_0)}{\left(\frac{\partial d(e_0,m_0)}{\partial e_0}\right)}$, and

(4.3.8′′) $\delta V'(q_1) = \frac{r'(m_0)}{\left(\frac{\partial d(e_0,m_0)}{\partial m_0}\right)}$.

A necessary condition for these equations to be satisfied is

(4.3.12) $$\frac{V'(q_0)}{V'(q_1)} = \frac{\delta(1-\delta)K'(q_0)}{\left(\frac{c'(e_0)}{\frac{\partial d(e_0,m_0)}{\partial e_0}}\right)} = \frac{\delta(1-\delta)K'(q_0)}{\left(\frac{r'(m_0)}{\frac{\partial d(e_0,m_0)}{\partial m_0}}\right)} .$$

If $q_0 > q^*$, then $K'(q_0) > K'(q^*)$. Also, if $q_0 > q^*$, we know (from the proof of Lemma 3.1) that $e_0 < e^*, m_0 < m^*$, $c'(e_0) < c'(e^*)$, and $r'(m_0) < r'(m^*)$. Since, by assumption

$$\frac{\delta(1-\delta)K'(q^*)}{\left(\frac{c'(e^*)}{\frac{\partial d(e^*,m^*)}{\partial e^*}}\right)} = \frac{\delta(1-\delta)K'(q^*)}{\left(\frac{r'(m^*)}{\frac{\partial d(e^*,m^*)}{\partial m^*}}\right)} = 1 \text{ and } \frac{\partial^2 d(e_t,m_t)}{\partial e_t \partial m_t} \geq 0,$$

we know $\frac{\delta(1-\delta)K'(q_0)}{\left(\frac{c'(e_0)}{\frac{\partial d(e_0,m_0)}{\partial e_0}}\right)} > 1$ and, from (4.3.12),

$\frac{V'(q_0)}{V'(q_1)} > 1$. But $q_0 > q_1$ implies $\frac{V'(q_0)}{V'(q_1)} < 1$ from the strict concavity of V. This is a contradiction. If $q_0 < q^*$, then $K'(q_0) < K'(q^*)$. Also, if $q_0 < q^*$, we know (from the proof of Lemma 3.1) that $e_0 > e^*, m_0 > m^*$, $c'(e_0) > c'(e^*)$, and $r'(m_0) > r'(m^*)$. Since, by assumption

$$\frac{\delta(1-\delta)K'(q^*)}{\left(\frac{c'(e^*)}{\frac{\partial d(e^*,m^*)}{\partial e^*}}\right)} = \frac{\delta(1-\delta)K'(q^*)}{\left(\frac{r'(m^*)}{\frac{\partial d(e^*,m^*)}{\partial m^*}}\right)} = 1,$$

we know $\frac{\delta(1-\delta)K'(q_0)}{\left(\frac{c'(e_0)}{\frac{\partial d(e_0,m_0)}{\partial e_0}}\right)} < 1$ and from (4.3.12)

$\frac{V'(q_0)}{V'(q_1)} < 1$. But $q_0 < q_1$ implies $\frac{V'(q_0)}{V'(q_1)} > 1$ from the strict concavity of V. This is a contradiction. Thus, $q_0^* = q^*$.

We have fully characterized the Pareto-efficient choice of quality, effort , and maintenance. We know $q_t^* = q^*$, $e_t^* = e^*$, and $m_t^* = m^*$, $t = 0,1,2,\ldots,\infty$. One way to think about these Pareto-efficient choices is that they are the equilibrium choices in a competitive market with perfect information. That is, if it were possible to contract on e and m, the equilibrium contract would specify e^* and m^* as the required choices and q^* would be the profit-maximizing entry quality level. Of course, before I can prove this, I must carefully define the properties which make up a competitive equilibrium in this model.

I will define equilibrium with respect to five properties and discuss each one briefly. Once equilibrium is defined, I will present a useful result, Proposition 4.1. I will then use this result to show that with perfect information, the Pareto-efficient choices will also be the competitive equilibrium choices. In Sections 4.3.4 and 4.3.5, I will again employ Proposition 4.1 to derive equilibrium choices under a variety of informational and cost structures.

Definition 4.1. The properties of a competive equilibrium are:

(1) equilibrium contracts must be feasible given the informational structure;
(2) tenants choose contracts and effort to maximize utility;
(3) the market "clears";
(4) landlords offer contracts,choose initial quality, and choose maintenance to maximize profits; and
(5) there is "free" entry by landlords.

In this model, landlords offer contracts which specify an initial (beginning-of-period) quality, and induce a combination of effort and maintenance, (e_t,m_t). Property (1), the feasibility requirement, regards the ability to contract on (e_t,m_t). For instance, if there is full (perfect) information, any (e_t,m_t) are feasible. However, if, as in Section 4.3.4, (e_t,m_t) are not observable and end-of-period transfers are not permitted, then the only feasible contracts have $e_t = 0$ and

$m_t \geq 0$. If end-of-period transfers are permitted so that simple damage deposits can be used, as in Section 4.3.5, then the set of feasible contracts has either $e_t = 0$ and $m_t \geq 0$, or $e_t \geq 0$ and $m_t = 0$. The connection between the informational structure, damage deposits, and feasibility in each case will be discussed in more detail in the next two sections.

Property (2) simply says that, in equilibrium, tenants choose contracts, taking the terms as given, to maximize utility each period.

Rent, one component of rental contracts, adjusts to "clear" the market. This, property (3), along with property (2), means that if the rent is too low, then tenant excess demand for that quality will bid the rent up. If the rent is too high, tenants would desire units of another quality level where they can obtain higher utility, causing the rent to fall. That is, conditions (2) and (3) imply that, in equilibrium, contracts must make tenants indifferent between different quality levels.

Landlords, as utility-takers, choose initial quality and the terms of the contract each period to maximize profits. This is property (4). Note that when combined with properties (2) and (3), utility-taking is equivalent to price/rent-taking. That is, given any quality, there is a maximum effective rent, $R_t + c(e_t)$, they can induce from the tenant.

Finally, property (5), free entry by landlords, implies that profits will equal zero in equilibrium. Note that this does not mean profits each period cannot be positive or negative, but that the discounted sum of the rent earned over time must equal the discounted sum of landlord costs over time.

Now that the properties of a competitive equilibrium have been defined and discussed, I will present a result, Proposition 4.1, which can assist in deriving a competitive equilibrium under various assumptions regarding the informational structure, use of damage deposits, and costs.

Proposition 4.1. In a competitive equilibrium, landlords choose contracts, from the set of feasible contracts determined by the informational constraints, to maximize social welfare and all of the surplus goes to the tenants.

Proof. Properties and (2) and (3) imply that $U(q_t) - R_t - c(e_t) = \Gamma$ for all q_t that might result. Given q_t, if the rent is too low, i.e. $U(q_t) - R_t - c(e_t) > \Gamma$, then tenant excess demand for that quality will bid the rent up. If the rent is too high, i.e. $U(q_t) - R_t - c(e_t) < \Gamma$, tenants would desire units of another quality level where they can obtain higher utility, causing the rent to fall. The landlord profit maximization problem can be written as

$$\max_{(q_0,e_t,m_t)} \left\{ \sum_{t=0}^{\infty} \delta^t \left[R_t(q_t) - r(m_t) - (1-\delta)K(q_0)\right] \right\}, \quad t = 0,1,2, \ldots, \infty,$$

subject to $U(q_t) - R_t - c(e_t) \geq \Gamma$, $q_{t+1} = q_t + d(e_t,m_t)$, and

(e_t,m_t) feasible (condition (1)).

Obviously, the inequality constraint will be binding, $U(q_t) - R_t - c(e_t) = \Gamma$. Substitute this into the objective function to get

$$\max_{(q_0,e_t,m_t)} \left\{ \sum_{t=0}^{\infty} \delta^t \left[U(q_t) - c(e_t) - r(m_t) - (1-\delta)K(q_0)\right] - \Gamma \right\},$$

$t = 0,1,2, \ldots, \infty$, subject to $q_{t+1} = q_t + d(e_t,m_t)$ and

(e_t,m_t) feasible (condition (1)).

But the solution to this problem is equivalent to the solution to the problem of maximizing social welfare subject to the feasibility constraint:

$$\max_{(q_0,e_t,m_t)} \left\{ \sum_{t=0}^{\infty} \delta^t \left[U(q_t) - c(e_t) - r(m_t) - (1-\delta)K(q_0) \right] \right\},$$

$t = 0,1,2, \ldots, \infty$, subject to $q_{t+1} = q_t + d(e_t,m_t)$ and

(e_t,m_t) feasible (condition (1)).

This proves the first part of Proposition 4.1. Condition (5) implies that tenants get all of the surplus, proving the second part of Proposition 4.1. Q.E.D.

Using Proposition 4.1, it is follows directly that, with perfect information where it is possible to contract on e_t and m_t, the equilibrium quality will be q^* and the contracts will call for effort and maintenance to be e^* and m^*, respectively.

Proposition 4.2. If $\delta(1-\delta)K'(q^*) = \dfrac{c'(e^*)}{\left(\dfrac{\partial d(e^*,m^*)}{\partial e}\right)} = \dfrac{r'(m^*)}{\left(\dfrac{\partial d(e^*,m^*)}{\partial m}\right)}$, then equilibrium would entail $q_t^* = q^*$, $e_t^* = e^*$, and $m_t^* = m^*$, $t = 0,1,2,\ldots,\infty$. The equilibrium rent would be $R^* = r(m^*) + (1-\delta)K(q^*)$ and the equilibrium per-period utility is $U(q^*) - c(e^*) - R^*$.

Proof. Proposition 4.2 follows directly from Proposition 4.1. Q.E.D.

In order to illustrate this result, it is useful to derive an iso-utility curve in rent-quality space. In particular, I would like to find the effective rent required to keep a tenant indifferent at various quality levels along the Pareto-efficient path. Remember that conditions (2) and (3) require that rents adjust to make tenants indifferent between any quality level in equilibrium. Although quality, in this case, begins at the steady-state level, it is still useful, for comparison, to derive this iso-utility curve.

Substituting the efficient effort and quality into the period t utility function (4.3.2) gives $U(q_t^*) - c(e^*(q_t^*)) - R_t = \Gamma$. Differentiating and rearranging, we get

$$\frac{dR_t}{dq_t^*} = U'(q_t^*) - c'(e^*(q_t^*))e^{*\prime} > 0.$$

In figure 4.1, this information about the slope of the iso-utility curve along the efficient path is used to draw a few such iso-utility curves. In this case, $U_1 < U_2 < U_3$.

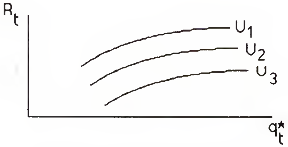


Figure 4.1. Iso-utility curves.

Drawing these iso-utility curves does not pin down what the equilibrium rent will be. We need to know more about the nature of competition to do that. I assume that competition is for tenants, so that landlords can earn only zero profit/utility and the tenants extract all the surplus in equilibrium (condition (5)). Under these assumptions, the discounted sum of the rent to the landlord must equal the discounted sum of the costs of initial quality and maintenance. This helps pin down the equilibrium utility, or rent as a function of quality. Finally, it must also be the case that profit maximizing behavior, taking utility or rent as given, on the part of landlords implies the choices of initial quality and maintenance made along the equilibrium path (condition (4)). The equilibrium choices must also satisfy the feasibility requirement (condition (1)), which depends on the informational structure. Only the Pareto-efficient choices of effort, maintenance, and quality satisfy all of these requirements according to Proposition 4.1.

4.3.4: Equilibrium Without Damage Deposits

In the next two sections, the informational structure of the model does not allow contracting on effort and maintenance. Although everyone can observe damages which have occurred by noting the change in quality, it is not possible for the courts to separate the effects of effort from maintenance. This is, of course, the reason why there is a problem of double moral hazard in this model.

In this section, for illustrative purposes, I make the assumption that the quality at the end of the period is not contractible in the current period. I do this by assuming no transfers can take place at the end of the period. This means that damage deposits are not allowed, since they may involve a transfer at the end of the period depending on the observed quality level. In the next section, I will allow end-of-period transfers, depending on observed quality, through simple damage deposit requirements.

One immediate result is that the tenant will always choose zero effort. This is because the tenant can never be punished for choosing inefficient effort. In other words, the inability to use damage deposits restricts the set of feasible contracts to those which induce $e_t = 0$ and $m_t \geq 0$, for any initial quality. Using Proposition 4.1, I can characterize equilibrium without damage deposits by solving the maximization problem

$$\text{(4.3.13)} \qquad \max_{(q_0, m_t)} \left\{ \sum_{t=0}^{\infty} \delta^t \left(U(q_t) - r(m_t) - (1-\delta)K(q_0) \right) \right\}$$

$$\text{subject to } q_{t+1} = q_t + d(0, m_t),\ q_0 > 0,\ t = 0, 1, 2, \ldots \infty.$$

Notice that the solution to this problem is merely the efficient outcome subject to the restriction that $e_t = 0$, $t = 0,1,2,\ldots,\infty$. I shall call this the second-best outcome because of the constraint.

To characterize the solution to (4.3.13), I will proceed as in the previous section. First, using properties of the optimal value function, I will prove that quality converges to a unique steady-state level, $\hat{q}$, from any $q_0 > 0$. In this proof it is also shown that $\hat{q}_{t-1} > \hat{q}_t \geq \hat{q}$ and $\hat{m}_{t-1} < \hat{m}_t \leq \hat{m}$, $t = 1,2,\ldots,\infty$,where.$\hat{m}$ and $\hat{q}$ are the steady-state values for maintenance and quality. Second, I will show that $\delta(1-\delta)K'(q^*) = \dfrac{r'(m^*)}{\left(\dfrac{\partial d(e^*,m^*)}{\partial m}\right)}$ implies $\hat{q}_0 > \hat{q}$. That is, the second-best initial quality is above the steady-state level.

Assuming $q_0 > 0$ given, the second-best path of maintenance and quality can be derived. Bellman's equation can be written as

$$V(q_t) = \max_{m_t} \left\{(U(q_t) - r(m_t) - (1-\delta)K(q_0)) + \delta V(q_{t+1})\right\} \tag{4.3.14}$$

where $q_{t+1} = q_t + d(0, m_t)$ and $q_0 > 0$ given.

The first-order condition for the right-hand side maximization problem is

$$-r'(e_t) + \delta V'(q_{t+1}) \frac{\partial d(0,m_t)}{\partial m_t} = 0. \tag{4.3.15}$$

and the equation of Benveniste and Scheinkman is

$$V'(q_t) = U'(q_t) + \delta V'(q_{t+1}). \tag{4.3.16}$$

Again, there exists a unique strictly concave V satisfying the functional equation (4.3.14) which solves (4.3.13). (See Appendix.) Furthermore, we know there exists a unique and time invariant optimal policy function $\hat{m}(\hat{q}_t)$. (See Appendix.) Substituting this optimal policy function into the transition equation determines the optimal path of the state variable, $\hat{q}_{t+1} = \hat{q}_t + d(0,\hat{m}(q_t))$. The derivative of this optimal transition equation can help determine whether or not there exists a unique steady-state in this equilibrium. This derivative is

(4.3.17) $$\frac{d\hat{q}_{t+1}}{d\hat{q}_t} = 1 + \frac{\partial d(0,\hat{m}_t)}{\partial m}\hat{m}'(q_t) .$$

Using an approach similar to that in the previous chapter, I can show

$0 \leq \frac{d\hat{q}_{t+1}}{d\hat{q}_t} < 1.$

Assume $\hat{q}_t^1 \geq \hat{q}_t^2$ and $\hat{q}_{t+1}^1 < \hat{q}_{t+1}^2$. From V continuous and strictly concave we have $V'(\hat{q}_{t+1}^1) > V'(\hat{q}_{t+1}^2)$. But this implies

$$\frac{r'(\hat{m}(\hat{q}_t^1))}{\delta\left(\frac{\partial d(0,\hat{m}(\hat{q}_t^1))}{\partial m_t}\right)} > \frac{r'(\hat{m}(\hat{q}_t^2))}{\delta\left(\frac{\partial d(0,\hat{m}(\hat{q}_t^2))}{\partial m_t}\right)}$$. From the strict convexity of r(m) and the concavity of d, the above inequality implies $\hat{m}(\hat{q}_t^1) > \hat{m}(\hat{q}_t^2)$ and $\hat{q}_{t+1}^1 > \hat{q}_{t+1}^2$, a contradiction. Show $\frac{d\hat{q}_{t+1}}{d\hat{q}_t} < 1$ by direct proof. Differentiating the FOC (4.3.15) gives $-r''\hat{m}'(q_t) + \delta V'\frac{\partial^2 d}{\partial m_t^2}\hat{m}'(q_t) + \delta V''\frac{\partial d}{\partial m_t}[1 + \frac{\partial d}{\partial m_t}\hat{m}'(q_t)] = 0.$

From (4.3.17), $1 + \frac{\partial d}{\partial m_t}\hat{m}'(q_t) = \frac{d\hat{q}_{t+1}}{d\hat{q}_t}$. Rearranging to solve for $\hat{m}'(q_t)$ gives

$$\hat{m}'(q_t) = \frac{-\delta V''\frac{\partial d}{\partial m_t}\frac{d\hat{q}_{t+1}}{d\hat{q}_t}}{\delta V'\frac{\partial^2 d}{\partial m_t^2} - r''}$$. Since we know $V' > 0$, $V'' < 0$, and $0 < \delta < 1$, $\hat{m}'(q_t) < 0$. Thus, from (4.3.17), $\frac{d\hat{q}_{t+1}}{d\hat{q}_t} < 1$.

Using this result and a phase diagram, as in figure 3.1, it is easy to verify that there exists a unique $\hat{q}$ such that $\hat{q} = \hat{q} + d(0,\hat{m}(\hat{q}))$ for any $\hat{q}_0$. We also get a little more information from the result that $\hat{m}'(q_t) < 0$. If $\hat{q}_0 > \hat{q}$, then as time passes quality must be decreasing as maintenance increases. The interpretation of this is that if, $\hat{q}_0 > \hat{q}$, the landlord lets $\hat{m}(q_0)$ be relatively small with $d < 0$. That is, he lets quality decline. But as quality declines he increases his maintenance to make quality fall by less and less each period. Of course, as $t \to \infty$, $\hat{m}(q_t) \to \hat{m}(\hat{q})$ since $q_t \to \hat{q}$. If $\hat{q}_0 < \hat{q}$, the landlord initially puts forth great maintenance to cause quality to rise, and then reduces maintenance to allow quality to rise by less each period.

Now that the path towards $\hat{q}$ has been summarized, we can solve for the efficient choice of initial quality, $\hat{q}_0$. Note that $\delta(1-\delta)K'(q^*) = \dfrac{r'(m^*)}{\left(\dfrac{\partial d(e^*,m^*)}{\partial m}\right)}$ implies $\delta(1-\delta)K'(\hat{q}) < \dfrac{r'(\hat{m})}{\left(\dfrac{\partial d(0,\hat{m})}{\partial m}\right)}$, since $\hat{m} > m^*$ (from $d(e^*,m^*) = d(0,\hat{m}) = 0$), $\hat{q} < q^*$ (or else $\hat{q}$ would be Pareto-efficient), $r'' > 0$, and $\dfrac{\partial^2 d(e_t,m_t)}{\partial e_t \partial m_t} \geq 0$. In this case, I will prove that the efficient initial quality is greater than $\hat{q}$. That is, efficiency calls for initial quality to be built above the steady-state level.

I shall prove that in the more general problem where the second-best initial quality is endogenous, $\hat{q}_0 > \hat{q}$. Bellman's equation can be written as

$$V(q_t) = \max_{q_0\, m_t} \{(U(q_t) - r(m_t) - (1-\delta)K(q_0)) + \delta V(q_{t+1})\} \tag{4.3.18}$$

where $q_{t+1} = q_t + d(0, m_t)$ and $q_0 > 0$.

The first-order conditions for this maximization are

$$U'(q_t) - (1-\delta)K'(q_0) + \delta V(q_{t+1}) = 0 \text{ and} \tag{4.3.19}$$

$$-r'(e_t) + \delta V'(q_{t+1}) \frac{\partial d(0,m_t)}{\partial m_t} = 0. \tag{4.3.20}$$

The equation of Benveniste and Scheinkman is

$$V'(q_t) = U'(q_t) + \delta V'(q_{t+1}). \tag{4.3.21}$$

To show that $\hat{q}_0 > \hat{q}$, begin by substituting (4.3.21) into (4.3.19) and rearranging (4.3.19) – (4.3.20) to get

(4.3.19′) $V'(q_t) = (1-\delta)K'(q_0)$ and

(4.3.20′) $\delta V'(q_{t+1}) = \dfrac{r'(m_t)}{\left(\dfrac{\partial d(0,m_t)}{\partial m_t}\right)}$.

Evaluating these equations at period zero gives

(4.3.6′′) $V'(q_0) = (1-\delta)K'(q_0)$ and

(4.3.8′′) $\delta V'(q_1) = \dfrac{r'(m_0)}{\left(\dfrac{\partial d(0,m_0)}{\partial m_0}\right)}$.

A necessary condition for these equations to be satisfied is

(4.3.22) $\dfrac{V'(q_0)}{V'(q_1)} = \dfrac{\delta(1-\delta)K'(q_0)}{\left(\dfrac{r'(m_0)}{\dfrac{\partial d(e_0,m_0)}{\partial m_0}}\right)}$.

If $\hat{q}_0 \le \hat{q}$, then $K'(\hat{q}_0) \le K'(\hat{q})$. Also, if $\hat{q}_0 \le \hat{q}$, we know $\hat{m}_0 \ge \hat{m}$ and $r'(\hat{m}_0) \ge r'(\hat{m})$.

Since $\dfrac{(1-\delta)K'(\hat{q})}{\left(\dfrac{r'(\hat{m})}{\dfrac{\partial d(0,\hat{m})}{\partial \hat{m}}}\right)} < 1$, we know $\dfrac{(1-\delta)K'(\hat{q}_0)}{\left(\dfrac{r'(\hat{m}_0)}{\dfrac{\partial d(0,\hat{m}_0)}{\partial \hat{m}_0}}\right)} < 1$ and, from (4.3.22),

$\dfrac{V'(q_0)}{V'(q_1)} < 1$. But $\hat{q}_0 \le \hat{q}$. implies $\dfrac{V'(q_0)}{V'(q_1)} \ge 1$ from the strict concavity of V. This is a contradiction. Thus, $\hat{q}_0 > \hat{q}$.

Using Proposition 4.1, it follows that if damage deposits are not allowed, given our informational structure, equilibrium will occur with $e_t = 0$ and $m_t = \hat{m}_t$, $t=0,1,2,\ldots,\infty$. That is, the second-best efficient outcome described above characterizes equilibrium when damage deposits cannot be used.

Proposition 4.3. When damage deposits are not allowed, then $e_t = 0$ and the equilibrium choices of quality and maintenance are $\hat{q}_t$ and $\hat{m}_t$, $t=0,1,2,\ldots,\infty$.

Proof. Propostion 4.3 follows directly from Propositon 4.1. Q.E.D.

4.3.5: Equilibria With Damage Deposits

In this section, I allow the end-of-period quality to be contractible through the use of simple discrete damage deposits of the form described in Section 4.2.5. Remember, the informational structure still does not allow contracting on effort and maintenance. We will see that there are two possible equilibria in this case. Which one obtains depends on assumptions concerning the cost of effort relative to the cost of maintenance. If it is less costly to maintain any damage level with only effort than with only maintenance, equilibrium will occur with positive damage deposits. Equilibrium is more efficient than without damage deposits. However, if it is less costly to maintain any damage level with only maintenance than with only effort, damage deposits are ineffective. In particular, equilibrium will occur with zero effort and if damage deposits are positive, it necessitates an equivalent reduction in rent. Equilibrium is no more efficient than in the previous section. There are no gains from allowing simple damage deposits in this case.

Before proceeding, it is useful to characterize the second-best path for quality and effort under the assumption that maintenance is zero in each period. This problem can be written as

$$(4.3.23) \qquad \max_{(q_0,e_t)} \left\{ \sum_{t=0}^{\infty} \delta^t \left(U(q_t) - c(e_t) - (1-\delta)K(q_0)\right) \right\}$$

$$\text{subject to } q_{t+1} = q_t + d(e_t,0),\ q_0 > 0,\ t=0,1,2,\ldots\infty.$$

Again, it can be shown that we have convergence to a unique steady-state quality, $\overline{q}$, starting from any $q_0 > 0$. To do this we need only apply Lemma 3.2 from the previous chapter. Note that, since $\delta(1 - \delta)K'(q^*) = \frac{r'(m^*)}{\left(\frac{\partial d(e^*,m^*)}{\partial m}\right)}$ implies $\delta(1 - \delta)K'(\overline{q}) < \frac{c'(\overline{e})}{\left(\frac{\partial d(\overline{e},0)}{\partial e}\right)}$, where $\overline{q}$ and $\overline{e}$ are steady-state quality and effort, $\overline{q}_0 > \overline{q}$. That is, second-best efficiency entails initial quality being built above the steady-state level. So $\overline{q}_t > \overline{q}$ and $\overline{e}_t < \overline{e}$ (from the proof of Lemma 3.2), with $m_t = 0$, $t=0,1,2,\ldots,\infty$. The proof of this is analogous to the proof that $\hat{q}_0 > \hat{q}$ in the previous section and will not be presented.

Continuing to assume $m_t = 0$, $t=0,1,2,\ldots,\infty$, it is not difficult to show that the above choices of effort can be supported with the appropriate use of damage deposits. This is important because with the informational structure, and damage deposits, it is possible to design contracts to induce positive effort. In particular, if $q_t^T = \overline{q}_t$ and $D_t \geq c(\overline{e}_t)$, equilibrium will occur with $e_t = \overline{e}_t$, $\overline{q}_t = \overline{q}_t$, and $\gamma_t = 0$, $t=0,1,2,\ldots,\infty$. Remember that equilibrium requires the maximization of surplus.

Proposition 4.4. When damage deposits are allowed and $m_t = 0$ the equilibrium choices of quality and effort are $\overline{q}_t$ and $\overline{e}_t$, $t=0,1,2,\ldots,\infty$.

Proof. Propostion 4.4 follows directly from Proposition 4.1. Q.E.D.

Note that the above case where $m_t = 0$ is almost equivalent to to the model introduced in Section 4.2. In fact, with only minor modifications to the damage function in the transition equation, the model is identical. Thus, Proposition 4.4 can be applied to formalize the main result of Section 4.2.

Proposition 4.4 does not imply that effort will be positive and second-best efficient in equilibrium, since it may be more efficient to have landlords perform maintenance instead. When damage deposits are allowed, the set of feasible contracts can induce $e_t \geq 0$ and $m_t \geq 0$. However, due to the double moral hazard problem, both cannot be positive in equilibrium. The reason is that if contracts call for positive effort and maintenance, then landlords have an incentive to reduce maintenance enough to retain the deposit and increase profits. Tenants know this and will increase utility by putting forth zero effort. Thus, the only way a positive damage deposit requirement can work to induce positive effort is if landlords cannot cheat to retain the deposit, $m_t = 0$. Otherwise, tenant effort must equal zero each period. In this case, landlords will want to put forth positive maintenance. This is the essence of the double moral hazard problem.

Next, I will examine the implications of two different assumptions regarding the cost of effort relative to the cost of maintenance. I will show that if it is always more costly for the tenant than for the landlord to maintain quality, equilibrium will occur with zero effort each period and damage deposits are zero or positive and ineffective. However, if it is always less costly for the tenant to maintain quality, equilibrium will occur with zero maintenance and positive effort. In this case, damage deposits are positive, effective, and result in greater efficiency than when they are not allowed.

Define $\tilde{e}(m_t)$ implicitly by $d(\tilde{e}(m_t),0) = d(0,m_t)$. That is, $\tilde{e}(m_t)$ is the effort required, when maintenance is zero, to get the same damage level which occurs when maintenance is m_t and effort is zero. Notice that $\bar{e} = \tilde{e}(\hat{m})$. We know this because $d(0,\hat{m}) = 0$ and $d(\bar{e},0) = 0$.

Assume that $c(\tilde{e}(m_t)) > r(m_t)$, for all m_t. It is more expensive to maintain quality for the tenant than it is for the landlord. The tenant will choose zero effort and let the landlord maintain the unit along the second-best path.

Proposition 4.5. If $c(\tilde{e}(m_t)) > r(m_t)$, for all m_t, then equilibrium occurs with $e_t=0$, $m_t = \hat{m}_t$, and $q_t = \hat{q}_t$.

Proof. Remember that, from Proposition 4.1, equilibrium choices of effort, maintenance, and quality must maximize the discounted surplus. Let $\bar{e}_t = \tilde{e}(\bar{m}_t)$ where $d(\bar{e}_t,0) = d(0,\bar{m}_t)$. The maximum discounted surplus, when $m_t = 0$, is

$V(\bar{q}_0) = \sum_{t=0}^{\infty} \delta^t(U(\bar{q}_t) - c(\bar{e}_t) - (1-\delta)K(\bar{q}_0))$. Define

$V' = \sum_{t=0}^{\infty} \delta^t(U(\bar{q}_t) - r(\bar{m}_t) - (1-\delta)K(\bar{q}_0))$. Since $c(\tilde{e}(m_t)) > r(m_t)$, for all m_t,

$V(\bar{q}_0) < V'$. But $V(\hat{q}_0) = \sum_{t=0}^{\infty} \delta^t(U(\hat{q}_t) - r(\hat{m}_t) - (1-\delta)K(\hat{q}_0)) > V'$, since $\hat{m}_t$ and $\hat{q}_t$, $t=0,1,2,\ldots,\infty$, maximizes discounted surplus when $e_t = 0$. Thus $V(\hat{q}_0) > V(\bar{q}_0)$ and equilibrium will occur with $\hat{m}_t$ and $\hat{q}_t$, $t=0,1,2,\ldots,\infty$. Q.E.D.

The equilibrium contract may or may not involve damage deposits. In the former case, although damage deposits are positive, they require an equivalent reduction in rent. The tenant will always forfeit the deposit and everyone knows this. The deposit requirement is ineffective and does not improve on equilibrium without damage deposits.

Now assume $c(\tilde{e}(m_t)) < r(m_t)$, for all m_t. It is always more expensive for the landlord to maintain quality than for the tenant. Here, tenants will want to substitute effort for landlord maintenance to increase the available surplus. The only way to credibly accomplish this is through damage deposits. But the tenant will not accept damage deposits unless the landlord cannot cheat to retain it. This is possible only if maintenance is set equal to zero.

Proposition 4.6. If $c(\tilde{e}(m_t)) < r(m_t)$, for all m_t, then equilibrium occurs with $m_t=0$, $e_t = \bar{e}_t$, and $q_t = \bar{q}_t$, $t=0,1,2,\ldots,\infty$.

Proof. Remember that, from Proposition 4.1, equilibrium choices of effort, maintenance, and quality must maximize the discounted surplus.

$$V(\hat{q}_0) = \sum_{t=0}^{\infty} \delta^t(U(\hat{q}_t) - r(\hat{m}_t) - (1-\delta)K(\hat{q}_0)) \text{ and}$$

$$V'' = \sum_{t=0}^{\infty} \delta^t(U(\hat{q}_t) - c(\tilde{e}(\hat{m}_t)) - (1-\delta)K(\hat{q}_0)).$$ We know $V(\hat{q}_0) < V''$. But

$$V(\bar{q}_0) = \sum_{t=0}^{\infty} \delta^t(U(\bar{q}_t) - c(\bar{e}_t) - (1-\delta)K(\bar{q}_0)) > V'' > V(\hat{q}_0).$$ Thus, equilibrium occurs with $m_t=0$, $e_t = \bar{e}_t$, and $q_t = \bar{q}_t$, $t=0,1,2,\ldots,\infty$. Q.E.D.

Remember that the above choices of effort can be supported with the appropriate use of damage deposits. In particular, if $q_t^T = \bar{q}_t$ and $D_t \geq c(\bar{e}_t)$, equilibrium will occur with $e_t = \bar{e}_t$, $\bar{q}_t = \bar{q}_t$, and $\gamma_t = 0$, $t=0,1,2,\ldots,\infty$. The tenant will always choose $e_t = \bar{e}_t$ and will never lose the deposit. The result is that the tenant chooses damage deposits in order to increase the available surplus. In this case, deposits play a positive and effective role in improving equilibrium over what would occur if they were not allowed.

It is somewhat surprising that in equilibrium with positive damage deposits, landlords do not perform any maintenance. In fact, even though tenant effort is too low to prevent the deterioration of quality over time, landlords do not put forth positive maintenance. With damage deposits to control the tenant moral hazard problem, one would expect the landlord to perform at least some maintenance. Of course, we know the reason for this result is that the informational structure results in a problem of double moral hazard. Unless maintenance is zero tenants will not put forth second-best efficient effort.

Notice that with or without positive damage deposits in equilibrium, quality deteriorates over time. The equilibrium requirement that tenants must be indifferent between quality levels implies that rent falls over time as well. In either equilibrium the free entry condition implies that profits over the infinite horizon must equal zero. That is, the discounted sum of rents earned over time minus the discounted sum of landlord costs must equal zero. Along the equilibrium path for quality and maintenance, it is not difficult to show that landlord per period costs do not decrease over time. Let $\hat{C}_t = C(\hat{q}_0, \hat{m}_t) = r(\hat{m}_t) + (1 - \delta)K(\hat{q}_0)$ be the cost to the landlord in period t, in the equilibrium with zero effort. Since, along the equilibrium path, $\hat{m}_{t-1} < \hat{m}_t$, $\hat{C}_t$ increases over time. In the other equilibrium, per period costs are the same each period. Since rent falls, costs rise, and profits equal zero, per period profit is initially positive, but at some point become negative.

These results seem consistant with what is actually observed. Quality does deteriorate over time. Also, it is likely that per-period profits are initially positive, but at some point become negative.

4.4: Conclusions

In this chapter, the role of damage deposits in rental housing markets characterized by moral hazard was explored. When there is only tenant moral hazard, it was shown that it is possible to design rental housing contracts with simple discrete damage deposits to induce efficient choices by tenants.

However, when there is both tenant and landlord moral hazard, this is not possible. It was shown, however, that under certain conditions damage deposits can improve on equilibrium which would occur without them.

I did not examine continuous damage deposits, where the landlord may withhold only a fraction of the deposit. The results with more general damage deposits should be similar to those obtained here. It is clear that Pareto-efficiency will obtain in the tenant moral hazard case. In the double moral hazard case, Pareto-efficiency will not obtain, but there may be equilibria with positive effort and maintenance. Therefore, it seems that generalizing the model with respect to the damage deposit mechanism will not contribute significantly toward understanding their role in rental housing markets.

Perhaps the main results in this chapter concern the path of quality and rent over time. When there is double moral hazard quality deteriorates over time and rent falls over time. Per-period profits begin positive, but eventually become negative in a competitive equilibrium.

I believe that these results can be generalized in a model where there may exist units of varying quality at any point in time and where there may be some hierarchy of rental housing markets. For instance, there may be high-income, middle-income, and low-income classifications in rental housing markets. In this case, there may be some important implications concerning the process of "filtering", a term introduced in Section 1.2 of this dissertation.

The main results of this paper concerning the equilibrium choices of effort and maintenance when damage deposits are allowed and concerning the path of quality and rent over time seem to be consistant with what is observed in rental housing markets. These results should be empirically testable. However, this empirical investigation is reserved for future research.

CHAPTER 5
SUMMARY AND CONCLUSIONS

I began this dissertation by reviewing the relevant rental housing literature. I do this primarily to illustrate that the basic ideas underlying this dissertation have not yet been adequately addressed. The basic ideas are that (1) there many informational problems which can lead to problems of adverse selection and moral hazard in rental housing markets and (2) tenant decisions, in the context of these informational problems, as well as landlord decisions, affect the equilibrium quality in rental housing markets. The main contribution of the models presented in this dissertation is that they help us understand the effects of tenant effort and the role of information, as well as the effects of landlord maintenance, in rental housing markets.

In Chapter 1, I also presented a simple introduction to the problems of adverse selection and moral hazard. The purpose of this introduction is to provide a basic foundation, which I build upon in the three original models presented in this dissertation.

In this dissertation, I introduced three models of behavior in rental housing markets. In each model, the choices of tenants and their effect on quality were important in equilibrium. The reason why their choices are important is that, in each model, their exists some informational asymmetry. In Chapter 2, landlords did not know a priori the transactions costs of moving for each tenant which may occupy the unit. In the models of Chapters 3 and 4, landlord maintenance and tenant effort, although known to each, were not contractible. The consequence of the informational problem was, in each of the three models, that Pareto-efficiency cannot obtain with a known finite horizon. For example, in the static model of Chapter 2 and in the finite-horizon model of Chapter 3 equilibrium is inefficient. However, in Chapter 3, the model is an infinite-horizon one where it is shown that it is possible to obtain Pareto-efficient choices in a subgame-perfect equilibrium.

In Chapter 2, tenants possess private information about their transactions costs of moving. The result is that when Nash equilibrium obtains, it must separate tenants who are good, in that they exert effort to maintain quality, from those who are bad, in that they exert no effort to maintain quality. In fact, in this model, good tenants are always worse off, in equilibrium, as a result of this informational problem than they would be in a first-best perfect information world. The way in which they are worse off is that quality is rationed to them. Quality rationing occurs when landlords quote an implicit price per unit of quality then proceed to offer a lower level of quality than that demanded by the tenant.

The main results of Chapter 2 are Propositions 2.1 – 2.5. In Proposition 2.1, I prove that the standard “market-clearing” contract will not be a Nash equilibrium in this informationally-constrained model. Proposition 2.3 simply establishes that a Nash pooling equilibrium does not exist. I briefly discuss the possibility of a pooling equilibrium by appealing the an alternative to the Nash definition. Propositions 2.2 and 2.4 provide the main result that when a Nash equilibrium exists, it must entail quality rationing to good tenants. The existence of a Nash equilibrium is then discussed in Proposition 2.5.

In Chapter 3, the asymmetric information problem takes the form of double moral hazard. After an exhaustive survey of the double moral hazard and dynamic games literature, I present a time-dependent supergame model of double moral hazard in rental housing markets. The tenant chooses effort and the landlord chooses maintenance. The problem is that in any period t, the choice of the tenant cannot be observed by the landlord and the choice of the landlord cannot be observed by the tenant. At the end of period t, the effects of tenant effort and landlord maintenance are summarized by the damage function. However, it is not possible for the courts to attribute damages to either inefficient effort or maintenance. If the relationship between the landlord and tenant is allowed to continue over an infinite horizon, I show in Proposition 3.1 that there exists a very simple discrete rental contract, contingent upon the history of quality, which supports Pareto efficiency in a subgame perfect equilibrium. This is accomplished through the use of "trigger" strategies not independent of the history of quality and, hence, the past choices of both the tenant and landlord. I also discuss the multiplicity of equilibria and possible extensions to include potential renegotiation over time. I also argue that, if there is a known finite probability that the relationship will end in each period, so that the relationship ends with probability one, Proposition 3.1 continues to hold. It is also demonstrated, using backwards induction, that if the tenant-landlord relationship lasts a known finite number of periods, Pareto-efficient choices cannot obtain. This lead me to consider equilibria in a model with double moral hazard and a known finite horizon for the landlord-tenant relationship.

In Chapter 4, the relationship between the tenant and landlord is assumed to last only one period. With only tenant moral hazard, I show that simple discrete damage deposits develop to preserve efficiency in equilibrium. When the model is extended to include double moral hazard, as expected in light of Chapter 3, Pareto efficiency cannot obtain. However, under certain conditions regarding the relative cost of effort versus maintenance, it is established that damage deposits can play a positive and effective role in supporting a more efficient equilibrium than would otherwise occur. This main result is established in Proposition 4.6.

Notice that the double moral hazard model of Chapter 4 is a natural extension of the model presented in Chapter 3. The informational structure remains very similar. In both chapters, although the timing is a little different, quality is observable. However, it is not possible to verify the actual choices by the landlord and tenant. This makes maintenance and effort not contractible. In each model, it is admitted that it is not possible to induce Pareto-efficient choices in equilibrium in a model with a known finite horizon . However, in Chapter 3 I show that when the relationship between the tenant and landlord is allowed to extend indefinitely, Pareto-efficient choices will obtain with the appropriate choice of contracts. Also, even though efficient choices will not obtain in equilibrium when the relationship lasts only one period, in Chapter 4 I show that under certain conditions, using very simple damage deposit contracts can lead to equilibrium with greater efficiency than without damage deposits.

In Chapter 3, most attention was devoted to analyzing behavior with respect to the steady-state. It was useful to assume that initial quality was the same as the Pareto-efficient steady-state quality in order to examine what happens as we move off the Pareto-efficient path. In Chapter 4, the model does not restrict attention to steady-state levels and the initial quality is endogenously determined. This may help us gain a better understanding of the way quality adjusts in competitive rental housing markets.

Perhaps the main results in Chapter 4 concern the equilibrium path of quality and rent over time. In this model, with or without positive damage deposits, in equilibrium, quality will deteriorate over time. In definition 4.1, I describe the properties of a competitive equilibrium in this model. The equilibrium requirement that tenants must be indifferent between quality levels implies that rent falls as quality falls over time. In either equilibrium, the free entry condition implies that profits over the infinite horizon must equal zero. That is, the discounted sum of rents earned over time minus the discounted sum of landlord costs must equal zero. Thus, per period profit is initially positive, but at some point in time becomes negative.

These results seem consistent with what is actually observed. Quality does deteriorate over time, even though with appropriate effort and/or maintenance it does not necessarily have to. Also, it is likely that per-period profits are initially positive, but at some point become negative.

Although the models presented in this dissertation do contribute towards understanding some of the informational problems associated with the rental housing markets, there are many issues and concerns which deserve more theoretical research before the role of informational problems can be fully understood. In particular, many of the unique characteristics of rental housing markets, for instance, the heterogeneity of rental units, have not been fully incorporated into the models of this dissertation. This heterogeneity between rental units generally implies the need for costly search. The models I have introduced do not address the problems associated with search. I have also not addressed the implications of tenants who may differ with respect to their utility functions. I suspect that incorporating these unique characteristics in models, where the informational structure of the models I have presented is retained, will not change the basic results. However, it may help us understand the observation that rental housing markets are often characterized by non-market-clearing.

Not only may extensions along the lines of incorporating additional unique characteristics of rental housing markets be desirable, but also the basic models may be generalized. For instance, some of the pseudo-dynamics of

Chapter 2 may be more fully developed in a dynamic model of adverse selection. Also, the results of Chapters 3 and 4 may be generalized with continuous rental functions and damage deposits.

In conclusion, I have sought to analyze and model behavior in rental housing markets characterized by informational asymmetries, which may lead to problems of adverse selection or moral hazard. To do this, I have introduced three original models of rental housing markets. Each model has lead to interesting results concerning equilibrium, especially with respect to quality. These models may help us understand more clearly the implications of asymmetric information, tenant effort, and landlord maintenance for the quality of dwellings in rental housing markets.

APPENDIX

In this appendix, I redefine the problems of Chapters 3 and 4 more generally using terminology consistent with Stokey (1989). I then list some assumptions and useful theorems provided by Stokey (1989). Furthermore, I show that these assumptions are satisfied by restrictions in the models of Chapters 3 and 4. Thus, we can apply these theorems without reservation. Define the problem more generally as

$$\text{(a)} \qquad V(q_0) = \max_{q_{t+1} \in \Gamma(q_t)} \left\{ \sum_{t=0}^{\infty} \delta^t F(q_t, q_{t+1}) \right\}$$

$t = 0,1,2, \ldots, \infty$, $q_0 \in Q$ given.

Define the functional equation

$$\text{(b)} \qquad V(q) = \max_{q' \in \Gamma(q)} [F(q,q') + \delta V(q')],$$

where the primes denote next-period values. Let $Q = \{q \mid 0 \leq q \leq \infty\}$, a convex subset of $\Re$, where Q is the set of possible values for the state variable, q. The correspondence describing the feasibility constraints is $\Gamma: Q \to Q$. The graph of Γ is $A = \{(q,q') \in QxQ : q' \in \Gamma(q)\}$. The return function is $F: A \to \Re$. Finally, $\delta \geq 0$ is the discount factor.

I want to discover restrictions on Q, Γ, F, δ which imply the solution to (b) corresponds to the solution to (a), the V which solves (b) is unique and strictly concave and differentiable, and the optimal policy function $g(q) = q'$ is unique and time invariant.

Assumptions:

A.A.1 Q is a convex subset of $\Re^m$, and the correspondence Γ: Q→Q is nonemty, compact-valued, and continuous.

A.A.2 The function F: $A \rightarrow \Re$ is bounded and continuous, and $0 < \delta < 1$.

A.A.3 F is concave; that is, $F[\theta(q_1,q_1') + (1-\theta)(q_2,q_2')] \geq \theta F(q_1,q_1') + (1-\theta)(q_2,q_2')$ for every $(q_1,q_1'),(q_2,q_2') \in A$ and all $\theta \in (0,1)$.

A.A.4 Γ is convex in the sense that for any $0 \leq \theta \leq 1$ and q, $\tilde{q} \in Q$, $q' \in \Gamma(q)$ and $\tilde{q}' \in \Gamma(\tilde{q}) \Rightarrow \theta q' + (1-\theta)\tilde{q}' \in \Gamma[\theta q + (1-\theta)\tilde{q}]$.

A.A.5 F is continuously differentiable on the interior of A.

Stokey (1989) proves the following theorems:

Theorem A.1. (Combines Theorems 4.2-4.6, Stokey (1989), pp. 72-79) Let Q, Γ, F, and δ satisfy A.A.1 - A.A.2 The solution to (b) coincides exactly to the solution to (a) – in terms of both values and optimal plans. In particular, the unique bounded continuous V satisfying (b) is the value function satisfying (a).

Theorem A.2. (Theorem 4.8, Stokey (1989), p. 81) Let Q, Γ, F, and δ satisfy A.A.1-A.A.4; let V satisfy (b); and the optimal policy correspondence G: Q→Q satisfy

(c) $$G(q) = \{q' \in \Gamma(q): V(q) = F(q,q') + \delta V(q')\} .$$

Then V is strictly concave and G is a continuous single-valued function.

Theorem A.3. (Convergence of the policy functions; Theorem 4.9, Stokey (1989), p. 82) Let Q, Γ, F, and δ satisfy A.A.1-A.A.4, and let V and g satisfy (b) and (c). Let C´(Q) be the set of bounded, continuous, concave functions f: $Q \rightarrow \Re$, and let $V_0 \in C'(Q)$. Let $\{V_n, g_n)\}$ be defined by $V_{n+1} = TV_n$, n=0,1,2,..., and

$g_n(q) = \underset{q' \in \Gamma(q_t)}{\operatorname{argmax}}[F(q,q') + \delta V_n(q')]$. Then $g_n \rightarrow g$ pointwise. If Q is compact, then the convergence is uniform.

Theorem A.4. (Differentiability of the value function; Theorem 4.11, Stokey (1989), p. 85) Let Q, Γ, F, and δ satisfy A.A.1-A.A.5, and let V and g satisfy (b) and (c). If $g_0 \in$ int Q and $g(q_0) \in$ int $\Gamma(q_0)$, then V is continuously differentiable a q_0, with derivatives given by $V'(q_0) = F'[q_0, g(q_0)]$.

The assumptions of the model developed in chapter 3 imply that A.A.1-A.A.6 are satisfied. I show this below.

A.A.1 is satisfied. Q is a convex subset of $\Re$, and $\Gamma(q)$: $Q \rightarrow Q$ is defined by $q' \in \{q + d(0,0), q + d(\overline{e},\overline{m})\}$. This implies the set A is closed and for any bounded set $\hat{Q}$ a subset of Q, the set $\Gamma(\hat{Q})$ is bounded. This implies Γ is compact-valued and upper hemi-continuous. Thus, since Γ(q) is single-valued, we get Γ(q) is continuous.

A.A.2 is satisfied. F is continuous by assumption. Since F is a continuous mapping of a compact metric space Q in $\Re$, F(Q) is closed and bounded and F(q) is bounded. $0 < \delta < 1$ by assumption.

A.A.3 is satisfied. F is concave since U is concave, c is convex, and r is convex.

A.A.4 is satisfied. A4.8 is equivalent to assuming the set A is convex. Since A is the graph of a concave function which maps a convex set $Q \rightarrow Q$ a subset of $\Re$, then A is convex.

A.A.5 is satisfied. F is continuously differentiable on the interior of A by assumption.

Thus, I can apply Theorems A.1-A.4 to the problems developed in Chapters 3 without reservation. These arguments can be easily adapted to accommodate the models of Chapter 4. None of the arguments are affected, since $d(e_t)$ is concave and $K(q_t)$ is convex. Also, although there is a new tenant occupying the unit each period, all tenants are identical.

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